VOLATILITY INFORMATION TRADING AND ITS IMPLICATIONS FOR INFORMATION ASYMMETRY, OPTION SPREADS, AND IMPLIED VOLATILITY SKEW

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ABSTRACT

Information asymmetry is a critical element in today's financial markets. While asymmetric information related to directional information trading has been extensively studied in the existing literature, there is limited research and evidence on how volatility information trading impacts the options market. This dissertation studies, both theoretically and empirically, the behaviors of volatility information traders in options markets and the implications of their behaviors on information asymmetry and options pricing.

I develop a model in which investors can trade multiple option contracts with varying strikes under an asymmetric framework. I show that volatility information trading is more likely to occur in Out of The Money (OTM) options if the overall presence of informed traders is low or if the relative liquidity in OTM options is better than At The Money (ATM) options. Moreover, I show that due to the variation in implicit leverage embedded in the option contracts, the OTM option contract contains a higher volatility information risk than the ATM option contract in equilibrium. In addition, I show that this volatility information risk differential plays a central role in forming the spread structure within an option series with the same underlying asset. Finally, I show that the shape of implied volatility skew (smile) is jointly determined by

volatility uncertainty and heterogeneous information risk across the option contracts.

I empirically examine the implications of my theory using US equity options data, including two intra-day trade and quote datasets from the Chicago Board Option Exchange (CBOE). I estimate the *Volume-Synchronized Probability of Informed Trading* (VPIN) variable to measure the volatility information risk in the option market. I show that OTM contracts, on average, have a higher probability of information trading than ATM contracts. I also document that volatility risk explains a considerable proportion of the spread variations in the US equity options market. Finally, I provide evidence that the difference in information asymmetry across strike prices not only helps to explain the dynamics of implied volatility skew but also has a significant impact on the degree to which a change in historical volatility affects the shape of the implied volatility skew.

BIOGRAPHICAL SKETCH

Wei Quan (William) was born in China in 1986. After he spent 14 years of his childhood in Beijing, he lived and studied in New Zealand for 2 years before attending a boarding school near the border between Wales and England in the United Kingdom. Wei Quan graduated with a First Class Honours degree in Economics from the London School of Economics and Political Science in 2008.

Wei Quan joined the Department of Economics at Cornell University in 2008 to pursue his Ph.D. degree in Economics.

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Most importantly, I am eternally grateful for the support, understanding, encouragement, and unconditional love of my parents, my wife Yidi, and the rest of my family. You have been, and always will be, believers in my aspirations and the foundation of my life.

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CHAPTER 1 Introduction

The role of information asymmetry in financial market has always been an interesting topic for both academics and practitioners. The existence of a group of investors who possesses informational advantage is proven to have significant impact on the trading environment, market efficiency, as well as asset prices. However, while traditionally directional information trading—investors who trade on private information about whether the price of an asset will move up or down—has been the primary concerns of market participants; the more diverse evolution of financial markets has made it increasingly important to understand the impact of information trading associated with another dimension of price movement—volatility information trading. Unlike directional informed traders (who have a number of alternative markets to invest in such as stock and futures markets), informed investors who trade in the volatility of underlying assets use only the option markets. Over the years, many articles in the press as well as in the existing literature have discussed the scope and significance of volatility trading in option markets. For example, Natenberg (1990) discusses why the primary concern of many option traders is the volatility of the underlying asset. Additionally, Ni, Pan, and Poteshman (2008) provide several quotes from the media that attribute significant option market activity to volatility trading.

The existence of volatility traders has expanded the role of the option market from a purely derivative market, where investors trade for purposes such as hedging activities, to an independent trading vehicle that allows investors to acquire exposure to the volatility of underlying assets through positions in option contracts. There has been increasing recognition of the importance of understanding volatility information trading. Not only because volatility is an essential factor in traditional option pricing theories, it has also become a new motivation for options trading. Moreover, an in-depth understanding of the role of volatility trading can add to our knowledge of option pricing features as well as the market microstructure of option markets.

The notion of an informed volatility trader might refer to any volatility investors in the option market who either know, or are able to better predict the realization of the future volatility of assets based on their private resources. They can take advantage of this private information by trading option securities to earn abnormal profits. In fact, private volatility information may exist independent of inside information about the directional movement of an asset.[1] Recent empirical work by Ni, Pan, and Poteshman (2008) finds that their non-market maker's net demand derived from option trading volume can predict the future realized volatility—a clear indication of informed volatility traders' presence in the options market. Moreover, they also document that information asymmetry caused by informed volatility traders has a significant price impact on option prices and such impact intensifies on days leading up to an announcement day. In a market maker oriented trading environment, one of the most important sources of risk for option market makers that cannot be easily hedged away is volatility risk.[2] This differential of information knowledge regarding volatility creates an information asymmetry situation that is similar to any other financial market—an adverse selection problem that will affect the belief of market participants, and thus the securities prices.

How do informed volatility investors behave in option markets where they face many alternative securities with varying strike prices, and what are the exact implications that these behaviors have on information structures and option prices? Determining answers to these questions has been the motivation for this dissertation.

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[1] Cox and Rubinstein (1985) discuss the possibility that some corporate decisions would surely result in more volatile future cash flow, but the direction of its impact is uncertain.
[2] Although the directional movement of an underlying's price is also a possible risk, it is relatively easy to hedge away. However, volatility risk is much harder to be hedged continuously and efficiently.

To improve the current understanding of volatility information trading and its implications, I build a model that generates new theories for a number of topics in the option pricing and the option market microstructure literature. Specifically, the model utilizes an asymmetric information framework to characterize the strategic interactions between an informed volatility trader, a continuum of market maker, and a liquidity hedger. Through a Bayesian Nash partial equilibrium, my theory is able to predict the behavior of an informed volatility trader under various circumstances. It also predicts that the degree of information asymmetry will be higher in an Out of The Money (OTM) option contract than an At The Money (ATM) contract as an equilibrium outcome. In addition, the theory suggests that heterogeneous information risk also plays a critical role in determining the spread structure in the option markets. Furthermore, this model also attempts to contribute to the literature of implied volatility skew (smile) from the perspective of asymmetric information. My theory indicates that in addition to the possibility that the implied volatility smile is a result of asymmetric information about volatility, the slope of the smile is also significantly influenced by the heterogeneity in information asymmetry caused by an informed trader's behavior.

Of course, it is always interesting and important to investigate whether the predictions of these theories are consistent with real world data. To accomplish this, I examine several implications of the model by empirically testing a number of hypotheses using US equity option data based on proprietary datasets from the Chicago Board Option Exchange (CBOE). I find that the majority of my predictions are indeed consistent with the data.

The rest of this dissertation is organized as the following. In Chapter 2, I introduce the theoretical part of my dissertation. Section 2.1 includes a literature review on a number topics related to the model. The construction of the model and its predictions are outlined in Sections

2.2 through 2.4. In Chapter 3, I empirically examine the behavior of the volatility information trader and the structure of information asymmetry in the US equity option market. Chapter 4 presents the test on the relationship between volatility information trading and spread structures in the option market. Chapter 5 investigates the impact of heterogeneous information asymmetry on implied volatility skew, and Chapter 6 concludes the dissertation.

CHAPTER 2 Theoretical Model

2.1 ***Introduction***

Many established theories regarding asymmetric information in financial markets such Kyle (1985), Back (1993), and Easley and O'Hara (1987), just to name a few, certainly help us to understand why there should be a positive relationship between option's prices and the level of information asymmetry concerning volatility. However, the situation can be noticeably more complex for option markets in which informed volatility investors see a much greater number of alternatives than equity markets because of the multiple option contracts available for the same underlying asset. These option securities feature different characteristics, from time to maturity to strike prices. One of the most interesting parameters of options is the strike price. On one hand, cheaper and more Out of The Money (OTM)[3] options provide higher implicit leverage within the contract to the favor of informed investors. On the other hand, near the money or At The Money (ATM) option contracts tend to have improved liquidity and lower transaction costs. It is particularly interesting to investigate how informed investors allocate their capital among different option contracts in the face of these tradeoffs. If informed volatility traders ultimately prefer one type of contract to another, then this helps in predicting which option's price will be most affected under certain circumstances. Therefore, knowing precisely how informed volatility traders behave across different contracts is essential for a better understanding of how the prices of different options will be impacted due information asymmetry. Unfortunately, only limited theories and empirical evidence exist in this area. Easley, O'Hara, and Srinivas (1998) expand the conventional information asymmetry model to characterize the behavior of informed directional traders between the stock market and the options market. By allowing the informed

[3] Out of The Money options refer to those options that have zero value if executed immediately. So for Call options, these are options with a strike price that is greater than the current underlying price.

trader to choose which market to invest in given a set of market conditions governed by exogenous parameters, they find that an informed directional investor will more likely trade in an options market if greater leverage and/or better liquidity exist. Empirically, Kaul, Gaurman, Mahendrarajah and Zhang (2004) find evidence to support the idea that informed traders prefer better liquidity over implicit leverage. However, Chakraverty, Gulen, and Mayhew (2005) claim that the leverage effect could be the dominating factor, as their econometric approach finds that the information share is greatest for OTM options. One possible reason for these inconsistent results in this academic space is the lack of a theoretical model that specifically focuses on the informed volatility trader's investment behavior while considering the features that are unique to option markets. This paper develops a theory that sheds new light on this issue. By endogenizing the implicit leverage effect into the model, the behavior of informed investors will have a strategic effect on market beliefs and option prices in equilibrium. Moreover, the volatility information trader may simultaneously trade multiple option contracts, effectively relaxing the implicit restriction that the degree of information asymmetry is the same across different option contracts. I show that, in equilibrium, informed volatility traders will behave in such a way that the information asymmetry in OTM contracts is greater than the information asymmetry in ATM contracts of the same underlying asset.

In addition, this paper is also motivated by the incomplete and segmented explanations of the intra-sectional spread properties between option contracts. Because it has been clearly documented that informed volatility trading exists in option markets and has a significant price impact, it is natural to establish a connection between option spread features and information asymmetry. Conventional asymmetric information theories [Kyle (1985), Back (1993), Easley and O'Hara (1987), and Glosten and Milgrom (1985)] suggest that the bid-ask spread as well as the expensiveness of a financial security will be positively related to the degree of information asymmetry in a market where the market maker is required to provide liquidity. So logically, the relative degree of information asymmetry between different option contracts, which should be a function of the informed volatility investor's behavior in choosing contracts with various degrees of implicit leverage and the market maker's reaction, could be playing a critical role in determining the spread features in today's market. Therefore, I am going to use an asymmetric information approach to model and explain the spread structure in option markets in relation to the behavior of informed volatility traders. Specifically, I attempt to contribute to the literature by establishing a linkage between volatility information trading and spread structure, both within the same option series and across different underlying assets.

In today's option markets, practitioners are increasingly interested in the shape of the volatility smile and its dynamics over time. Another key objective of this model is to touch upon the literature of options pricing using an asymmetric information framework, thus shedding light on the ongoing discussions on the characteristics and dynamics of the implied volatility skew (smile) in option market.

As previously stated, much of the existing literature on asymmetric information in option markets focuses on the impact of information asymmetry about the mean of the underlying asset; in other words, the effect of informed directional investors on option markets. These studies include John, Koticha, and Subrahmanyam (1993), Back (1993), Biais, and Hillion (1994), Brennan and Cao (1996), and Easley, O'Hara, and Srinivas (1998). Cherian and Jarrow (1998) are the first to incorporate information asymmetry about the future volatility of an asset into the larger discussion of option pricing. They utilize an asymmetric information model in which informed directional and volatility traders can trade in an option market. They show that as a possible self-fulfilling equilibrium, option values as expectations of conditional Black-Scholes prices could cause the implied volatility to be different across option strikes. Nandi (1999) uses a multi-period framework to model the effect of asymmetric information among volatility traders. He shows that equilibrium option prices, and thus the level of implied volatility, depend on the net order flow in the market. Both of these papers generate the widely documented feature of the implied volatility smile. The same option pricing features can also be found in other streams of option pricing models such as stochastic volatility models [Hull and White (1987), Wiggins (1987), Stein and Stein (1991), Bates (1996), etc.], and models that include GARCH features [Heston (1993) and Heston and Nandi (1997)]. However, to the best of knowledge, none of these models systematically consider the effect of the volatility investors' strategic behavior across strike prices on implied volatility. In addition to being able to generate bid-ask predictions within the model, this model is distinct from other option pricing models in being able to introduce heterogeneity in the information structure across the option contracts through modeling the behavior of the volatility traders. The model adds an extra dimension to the existing explanations by freeing up the restriction that information asymmetry must be the same across option series of the same underlying asset. The possible heterogeneity in information asymmetry (and thus the shape of volatility smile) will be a function of the informed trader's investment strategy, in addition to other factors related to the market environment. I show that, in equilibrium, there are two essential aspects that jointly determine the shape of the implied volatility smile: the stochastic volatility (or volatility uncertainty) arising from the asymmetric information and the heterogeneous information risk across the strikes.

The rest of this chapter will be organized as the following: in Section 2.2 I will describe the model setup and assumptions used. The equilibrium of the model is defined in Section 2.3.

The results and implications for informed volatility trader's behavior, information asymmetry, option price, implied volatility smile, and bid-ask spread are presented and discussed throughout Sections 2.4.1 through 2.4.4.

2.2 *The Model*

The model builds on an asymmetric information structure that can be found in the extensive literature of market microstructure [Easley and O'Hara (1987) and Glosten and Milgrom (1985)] that assumes a strategic environment among market makers and investors. It considers an economy in which call option prices are determined in a partial equilibrium framework that was first introduced by Cherian and Jarrow (1998).

There are three players in the economy: a continuum of risk-neutral market makers, a profit-maximising informed investor with private information about volatility, and a liquidity hedger whose investment behaviors are exogenously determined. There is no specific risk preference assumed for the liquidity hedger, who is assumed to trade for exogenous purposes such as liquidity hedging requirement. The model consists of two periods; the number of periods is minimized to facilitate the understanding of the model. At time 0, all three players enter the market. They share the same information on the stock price S, which is known to everyone. The participants in the market face different information sets toward the true value of volatility $\sigma \in \{\overline{\sigma}, \underline{\sigma}\}$. Volatility information is available at time 0 to the informed investor as private information. At time 1, the true value of the volatility is revealed to all, and the option price converges to an equilibrium level, which is the discounted expected value of the option's future payoff according to a mutually agreed distribution on the stock price:

$$C_{t=1}(S, \sigma, k_i) = E\left[\max(S_T - k_i, 0) \,|\sigma \in \{\overline{\sigma}, \underline{\sigma}\}\right]e^{-r} \quad (1)$$

Notice that this expression is not an option price derived from any pricing formula, it is merely a discounted non-arbitrage option value that is determined by its expected future payoff using risk neutral probabilities. The time interval between Time 0 and Time 1 is assumed to be short enough that any potential interest earned during this period is minimal and close to zero.

The strategic interactions between the market participants will take place at time 0. Each market maker will set their bid (B_i) and ask (A_i) prices for the two option contracts conditional on the current stock price and the orders they receive; each market maker is allowed to quote one unit in quantity and prices are set in a competitive environment. Nature will choose which investor type to trade, with a probability α that the informed volatility trader is chosen. If an informed trader is selected, it is assumed that the informed investor's action at time 0 will be to BUY options if she sees a high volatility, and to SELL option if observes a low volatility. Moreover, the informed trader must determine with what probability ρ_i she will invest her total capital W^Iin each of the two contracts in order to maximize her expected profit. Therefore, her profit from investment in one share of contract-i will be $C_i(\overline{\sigma}) - A_i$. There is no explicit restriction on whether informed traders are allowed to borrow before entering the option market when exploiting their informational advantage. However, it is implicitly assumed here that if she decides to borrow, there is an upper limit to which she can expand her capital through borrowing; therefore, the total available capital for the informed trader is constrained at a fixed level (W^I). This assumption is an extremely relaxed one in the context of the entire model. The result of the model does not rely on the actual size of the investable wealth by the informed trader. As long as the informed face some financial frictions for borrowing (which tends to be the case in today's financial markets), there will be an upper limit on the amount of capital that an informed trader can invest, which is sufficient to guarantee the results. Finally, if an uninformed liquidity hedger is chosen, she will buy and sell options equal to the same total value of W^U. In addition, between the two available contracts, she will be assigned a probability h_i of the total value W^U by nature to trade in contract-i. When nature has decided whom to trade, the submitted orders will be channeled to an "order-pool" for each option contract, and then these pooled orders will be allocated randomly to the continuum of market makers. Thus, when the pooled trades are distributed randomly across the population of market makers, for each market maker, the ex-ante probability of receiving an order for contract-i is equal to the probability of the informed investor choosing contract-i. Therefore, ex-ante, the main objective of the market makers is to conjecture both the likelihood of each of their quotes being hit as well as under what circumstances each case will occur. At time 1, either $\overline{\sigma}$ or $\underline{\sigma}$ is realized to the public with equal probability. In the rest of this paper, the initial endowment of the two types of investors will be assumed to be the same. This can be achieved without the loss of generality: the relative wealth will only impact the priors of the market maker, yet the probability of who is selected to trade (α) is set to be arbitrary.

There are two call option contracts[4] traded in this economy: $C(S,\sigma,k_1)$ *and* $C(S,\sigma,k_2)$, which only differ in terms of their strike prices: $k_1 > k_2 \geq S$[5]. Because I am not specifying any pricing function for the options at this stage, the following properties on the true value of an option at time 1 are assumed:

$$C(S,\sigma,k_1) < C(S,\sigma,k_2) \tag{2}$$

$$\frac{C(S,\overline{\sigma},k_1) - C(S,\underline{\sigma},k_1)}{C(S,\underline{\sigma},k_1)} > \frac{C(S,\overline{\sigma},k_2) - C(S,\underline{\sigma},k_2)}{C(S,\underline{\sigma},k_2)} \tag{3}$$

The first assumption is very intuitive. Inequality (2) essentially states that the call option contract with the higher strike price (referred to hereinafter as Contract-1) will have a lower value, thus the potential of providing higher implicit leverage. The second assumption is

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[4] The choice of Call options is simply for the reason of convenient demonstration. One can easily apply the framework of this model into considering the trading of Put options where such a scenario is more relevant. All major qualitative conclusions from the model would remain unchanged.

[5] This model does not consider options that are In The Money (ITM) for two reasons. First, ITM contracts provide the least leverage to informed traders compared with OTM and ATM contracts. Second, future uncertainty on the value of an ITM contract is relatively small; including it into the model does not add much insight and will not change the qualitative result.

equivalent to saying that the elasticity of the option value with respect to volatility is smaller for the ATM contract with a lower strike price (referred to hereinafter as Contract-2) than the OTM contract. Notice that these assumptions are quite general and usually true in an actual option market. In fact, the first assumption on option price is always true under any non-arbitrage environment. Intuitively, the second assumption assures that the option values satisfy the condition that the rate of return, or the bang per buck, is greater for a single share of the more OTM Contract-1 than Contract-2. Alternatively, another way to interpret Inequality (3) is that it assumes the OTM has a higher per dollar exposure to volatility, thus providing higher implicit leverage for volatility traders. One could show that, under the Black-Scholes pricing formula, this inequality condition can be written as:

$$\frac{C(S,\overline{\sigma},k_1)-C\left(S,\underline{\sigma},k_1\right)}{\left(\overline{\sigma}-\underline{\sigma}\right)C\left(S,\underline{\sigma},k_1\right)}>\frac{C(S,\overline{\sigma},k_2)-C\left(S,\underline{\sigma},k_2\right)}{\left(\overline{\sigma}-\underline{\sigma}\right)C\left(S,\underline{\sigma},k_2\right)}$$

$$\Leftrightarrow \lim_{\overline{\sigma}\to\underline{\sigma}}\frac{C(S,\overline{\sigma},k_1)-C\left(S,\underline{\sigma},k_1\right)}{\left(\overline{\sigma}-\underline{\sigma}\right)C\left(S,\underline{\sigma},k_1\right)}>\lim_{\overline{\sigma}\to\underline{\sigma}}\frac{C(S,\overline{\sigma},k_2)-C\left(S,\underline{\sigma},k_2\right)}{\left(\overline{\sigma}-\underline{\sigma}\right)C\left(S,\underline{\sigma},k_2\right)}$$

$$\Leftrightarrow \frac{\frac{\partial C(S,\sigma,k_1)}{\partial\sigma}}{C\left(S,\underline{\sigma},k_1\right)}>\frac{\frac{\partial C(S,\sigma,k_2)}{\partial\sigma}}{C\left(S,\underline{\sigma},k_2\right)}$$

In terms of Black-Scholes Greeks,

$$\Leftrightarrow \frac{Vega(k_1)}{BS\left(S,\underline{\sigma},k_1\right)}>\frac{Vega(k_2)}{BS\left(S,\underline{\sigma},k_2\right)} \tag{4}$$

Thus, Inequality (4) implies that if the conditional B-S formula correctly characterizes the option value in the second period of this economy, then the second assumption requires the ratio of option Vega to its price, or 'the per-dollar Vega', in the low volatility state to be higher for the OTM Contract-1. Due to the complexity of analytically proving this, I performed a large number of numerical calculations using a wide range of parameter values to examine the validity of this assumption as shown in Inequality (4) .The results convincingly suggest that the assumption also appears to be highly plausible in a B-S setting. Figure 1 illustrates an example in the numerical examinations in which the stock price is $35. We can see that the implicit leverage (Vega/$) of options clearly increases with the strike price.

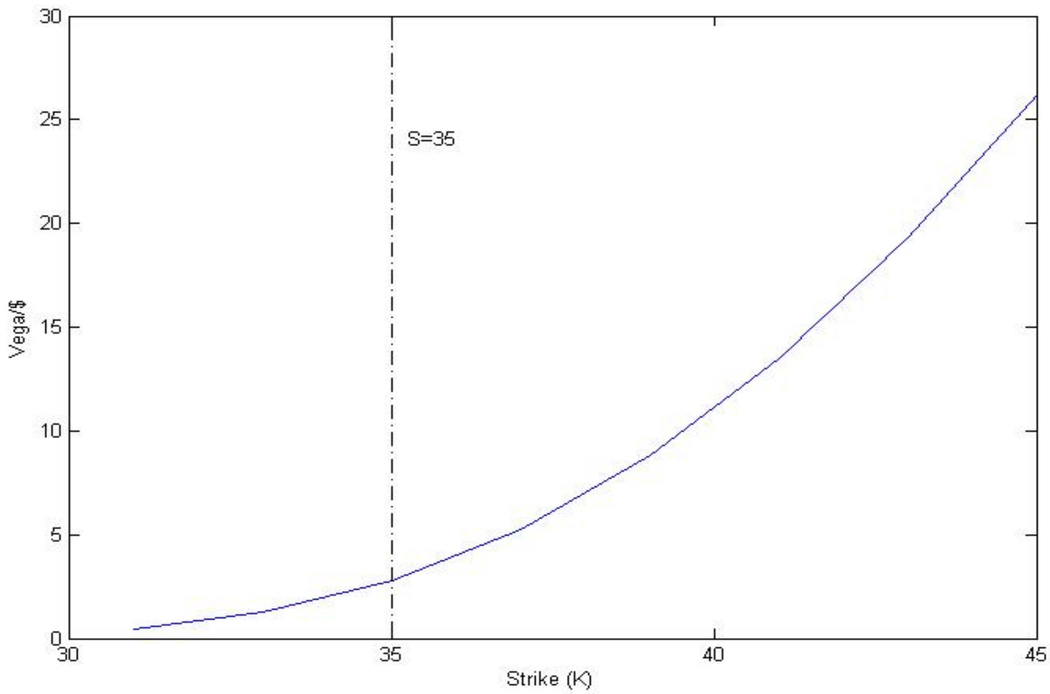

Figure 1. Implicit Leverage for Volatility Traders under B-S Formula

This figure demonstrates the varying implicit leverage, measured by option's Vega/option price, embedded in options with different strike under Black-Scholes pricing formula. Parameters used in this example are S=35, σ=0.3, r=0.3, τ (maturity in years)=0.125.

To summarize, in the economy described above, the market maker's and the informed investor's decision problem can be characterized as follows:[6]

Market Maker's problem:

At time 0, competitive market makers set their bid and ask so that the expected profit in each option contract is zero:

$$A_i = E_i[C(\sigma, k_i)|BUY_i]$$

$$= pr(\overline{\sigma}|BUY_i) * C_1(\overline{\sigma}, k_i) + pr(\underline{\sigma}|BUY_i) * C_1(\underline{\sigma}, k_i) \quad (5)$$

$$B_i = E_i[C(\sigma, k_i)|SELL_i]$$

$$= pr(\overline{\sigma}|SELL_i) * C_1(\overline{\sigma}, k_i) + pr(\underline{\sigma}|SELL_i) * C_1(\underline{\sigma}, k_i) \quad (6)$$

where $pr(\sigma|\quad)$is the conditional probability on the state of the economy from the market maker's perspective should one of their quotes gets hit by traders' orders.

Informed Investor's Problem:

An informed investor seeks to maximize her total expected profit using her informational advantage, and does so by allocating probabilities between the two option contracts. Let us first consider the case of a high volatility state, i.e., $\sigma = \overline{\sigma}$. Informed σ-trader needs to determine with what probability she wants to invest in each option contract by choosing ρ_1 and ρ_2:

$$\max_{\rho_i}\{\sum_i \rho_i^B \frac{W}{A_i}(\,C_i(\overline{\sigma}) - A_i)\}$$

$$s.t. \sum \rho_i^B = 1 \; and \; 0 \leq \rho_i^B \leq 1 \quad (7)$$

If a low volatility state is observed, i.e., $\sigma = \underline{\sigma}$, the informed investor will sell options to maximize her total expected profit:

[6] The behavior of the liquidity hedger is determined exogenously by nature.

$$\max_{\rho_i}\{\sum\nolimits_{i}\rho_i^s\frac{W}{B_i}(B_i-C_i(\underline{\sigma}))\}$$

$$s.t.\sum\rho_i^s=1\ and\ 0\leq\rho_i^S\leq1 \qquad (8)$$

Notice that the probabilities of choosing contract-i could be different in high or low volatility states. In this model, the ask or bid prices in equilibrium add an extra dimension to the existing leverage effect implied within the structure of the option contract. The higher the equilibrium ask (or bid) price as a result of information asymmetry; the lower the implicit leverage for the informed trader. This dynamic between the market maker and the informed investor generates an interesting trade-off. The informed volatility trader wants to increase her probabilistic strategy in Contract-1, ρ_1, as much as possible to take advantage of the implicit leverage and better rate of return; however, she also wants to reduce ρ_1 to hide her presence in the market from the market maker, thus holding down the equilibrium price of Contract-1. Therefore, the informed trader has to maximise her profit by balancing this trade-off given the available contracts. Finally, the second constraint in the informed investor's problem can also be interpreted as a "no-shorting" constraint after she entered the market, which restricts the investor from short-selling one contract for the sake of purchasing more of the other. However, as discussed earlier, this constraint is also consistent with the idea that borrowing outside the option market is implicitly allowed in this model.

It is useful to display this informational structure and trading process in diagrams:

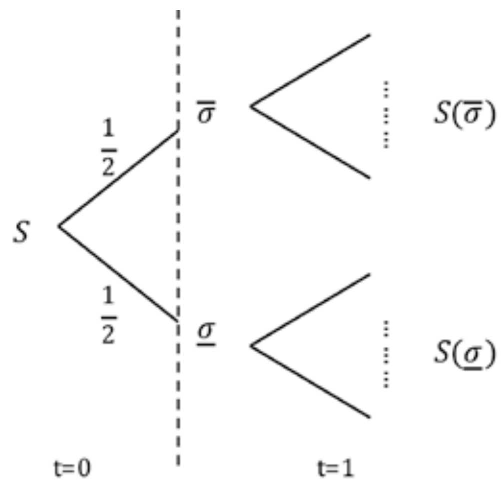

Figure 2. The Timeline of Underlying Asset's Price Movement.

This figure shows that the timeline of the underlying asset has two stages. Its initial value is known to the public in t=0, but not the volatility. In t=1, the volatility state is revealed to everyone and the asset's future price is distributed given the true volatility.

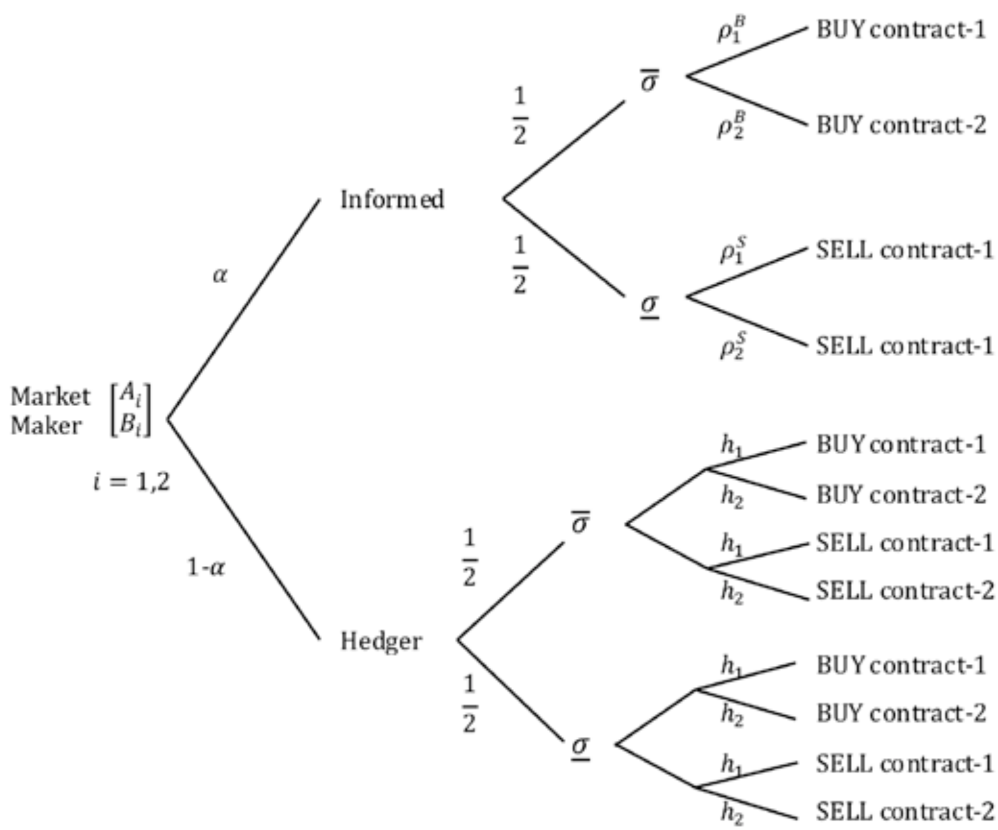

Figure 3. The Game Structure at Time 0.

This figure summarizes how trading orders are likely to arrive from the maker maker's perspective.

All of the players know the structure of the trading process. Market makers will associate a higher conditional probability to the high (low) volatility state if they receive a BUY (SELL) order because the informed investor will buy options only if she observes high volatility. Thus, with Bayes updating, the key objective for the market maker is to form her expectation of the option price at Time 0 conditional on which of her quotes might be hit. For example, from the market maker's perspective:

$$\Pr(\overline{\sigma}|BUY_i) = \frac{\Pr(\overline{\sigma}\ \&\ BUY_i)}{\Pr(\overline{\sigma}\ \&\ BUY_i) + \Pr\left(\underline{\sigma}\ \&\ BUY_i\right)}$$

$$where\ \ \Pr(\overline{\sigma}\ \&\ BUY_i) = \Pr(Informed\ BUY_i\ in\ \overline{\sigma}) + \Pr(Hedger\ BUY_i\ in\ \overline{\sigma})$$

$$= \frac{1}{2}\alpha\rho_i + \frac{1}{2}(1-\alpha)h_i$$

$$and\ \ \Pr\left(\underline{\sigma}\ \&\ BUY_i\right) = \Pr\left(Hedger\ BUY_i\ in\ \underline{\sigma}\right) = \frac{1}{2}(1-\alpha)h_i$$

$$Thus, \qquad \Pr(\overline{\sigma}|BUY_i) = \frac{\frac{1}{2}\alpha\rho_i + \frac{1}{2}(1-\alpha)h_i}{\frac{1}{2}\alpha\rho_i + (1-\alpha)h_i} = \frac{\alpha\rho_i + (1-\alpha)h_i}{\alpha\rho_i + 2(1-\alpha)h_i}$$

$$Similarly, \qquad \Pr(\overline{\sigma}|SELL_i) = \frac{(1-\alpha)h_i}{\alpha\rho_i + 2(1-\alpha)h_i}$$

One example for the duration of each period in the model could be one trading day. In this case, if one thinks of Time 0 as being trading day t, then Time 1 is the beginning of trading day t+1; therefore, the daily volatility and distribution of the closing stock price is determined at the opening of t+1.This example also implies that multiple strategic interactions between the market participants may occur during one trading day. Consequently, market makers are able to continuously update their beliefs throughout the day; however, these dynamic implications will not be discussed in this section.

A final remark needs to be made at this stage. One might argue that it is more realistic to assume a case of continuum of informed investors in the option market. In fact, there is an alternative interpretation of the model environment that will eventually lead to the same information (probability) structure and player's problem. This alternative structure could capture a market situation in which there is a single market maker, a continuum of informed volatility investors, and also a continuum of liquidity hedgers. Nature still plays the same role in deciding which volatility state is realized and in choosing whether informed investors would trade with probability α. However, either type of investors, if selected by the nature, must still choose which of the two option contracts to trade at Time 0—but not both. The liquidity hedgers will buy or sell with equal probability regardless of the volatility state, and nature will assign a probability h_i for trading Contract-i. But for the informed investors, each investor's strategy at the beginning of the game will be optimally choosing the probability ρ_i of trading either contract. Because ρ_i is a probability variable, both constraints $\sum \rho_i^B = 1 \; and \; 0 \leq \rho_i^B \leq 1$ are automatically satisfied. Finally, the risk-neutral market maker's objective remains to quote the bid and ask prices for each contract such that, given the informed investor's probabilistic strategy, the expected profit is zero. This model environment will produce exactly the same beliefs, objective functions, as well as equilibrium results as the previously explained environment. Both explanations have significant and valid economic interpretations. More importantly, both standpoints can lead to the same structural model that is designed to answer the fundamental question that this paper is seeking to explore: How does informed investors' behavior in option markets as a whole affect the information structure of the market and the properties of option prices?

2.3 *Equilibrium*

Time 1

The equilibrium prices of call options at Time 1 are determined solely by a non-arbitrage condition since there is no differential information remains in the market.[7] The value of the call options is equal to its future expected payoff conditional on the current stock price and the distribution given the realization of volatility. For example, if a log-normal distribution is assumed to be the common belief among market participants, then the equilibrium option value in this period can be priced by the Black-Scholes formula conditional on the realization of volatility. In fact, any arbitrary distributions could support such equilibrium; please refer to Cherian and Jarrow (1998) for a detailed discussion.

Time 0

Given the equilibrium prices in Time 1, the equilibrium at this stage is characterized as a Nash Equilibrium, such that neither the market maker nor the informed trader will deviate from their strategies.

Therefore, in equilibrium, given the model parameters $(\alpha, h_i, \overline{\sigma}, \underline{\sigma}, S, k_i)$, the informed trader's strategy set $\{\rho_i^{S^*}, \rho_i^{B^*}\}$, and the set of ask and bid prices from the market maker $\{A_i, B_i\}$:

The informed investor's expected profit is maximized given their information at Time 0.

The market maker earns zero expected profit at Time 0.

2.4 *Results*

I will discuss the result of this model for the most part considering the state of $\sigma = \overline{\sigma}$ in the second period. Most of the results will be symmetric when low volatility is observed and the

[7] This follows the same characterization of equilibrium as in Cherian and Jarrow (1998).

informed investor decides to sell options; this will be become intuitive and clear in the later sections where the implications for cross-sectional spread features are analyzed. Because some illustrations of the results involve numerical demonstrations, I am going to select specific values for the option characteristics. Unless otherwise stated, the following parameter values are assigned to each option contract as *Standard Example* in the numerical examples.

$$k_1 = 35; k_2 = 30;\ S = 30;\ T = 45; r = 0.03$$

$$and \quad \overline{\sigma} = 0.35;\ \underline{\sigma} = 0.25;$$

2.4.1 Informed Investor's Behavior & Information Asymmetry:

The process for obtaining equilibrium in this economy involves solving for the Nash equilibrium, which can be achieved through numerous methods. I will demonstrate the process with a simple and standard procedure.

In substituting the market maker's equilibrium strategy into the objective function of the informed investor, we obtain the new problem as below:

$$\max_{\rho_i,\rho_j} \frac{w\rho_i}{A_i}\left(C_i(\overline{\sigma}) - A_i\right) + \frac{w\rho_j}{A_j}\left(C_j(\overline{\sigma}) - A_j\right)$$

$$s.t.\ \ \rho_i + \rho_j = 1 \;\&\; 0 \le \rho_i \le 1$$

$$\Rightarrow\ Max_{\rho_i}\ \frac{w\rho_i \cdot \left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)(1-\alpha)h_i}{\alpha\rho_i + 2(1-\alpha)h_i} \cdot \frac{1}{A_i} + \frac{w\rho_j \cdot \left(C_j(\overline{\sigma}) - C_j(\underline{\sigma})\right)(1-\alpha)h_j}{\alpha\rho_j + 2(1-\alpha)h_j} \cdot \frac{1}{A_j}$$

$$s.t.\ \ \rho_i + \rho_j = 1 \;\&\; 0 \le \rho_i \le 1$$

$$\Rightarrow\ Max_{\rho_i}\ \frac{w\rho_i \cdot \left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)(1-\alpha)h_i}{\alpha\rho_i C_i(\overline{\sigma}) + (1-\alpha)h_i(C_i(\overline{\sigma}) + C_i(\underline{\sigma}))} + \frac{w\rho_j \cdot \left(C_j(\overline{\sigma}) - C_j(\underline{\sigma})\right)(1-\alpha)h_j}{\alpha\rho_j C_j(\overline{\sigma}) + (1-\alpha)h_j(C_j(\overline{\sigma}) + C_j(\underline{\sigma}))}$$

$$s.t.\ \ \rho_i + \rho_j = 1 \;\&\; 0 \le \rho_i \le 1$$

Because the ask price in the profit function for the informed trader will be a non-linear function of the objective variable in equilibrium, a quick check on the concavity of the profit function will provide some insights to the nature of the solution. Figure 4 plots the total profit of the informed trader against the increasing value of ρ_1 for a particular set of parameter values. It shows that the profit function first increases in ρ_1 and then decreases after reaching its peak. If different parameter values for the probabilities are used, the curve primarily shifts horizontally; different option contracts will alter the slope of this curve, but its shape remains the same.

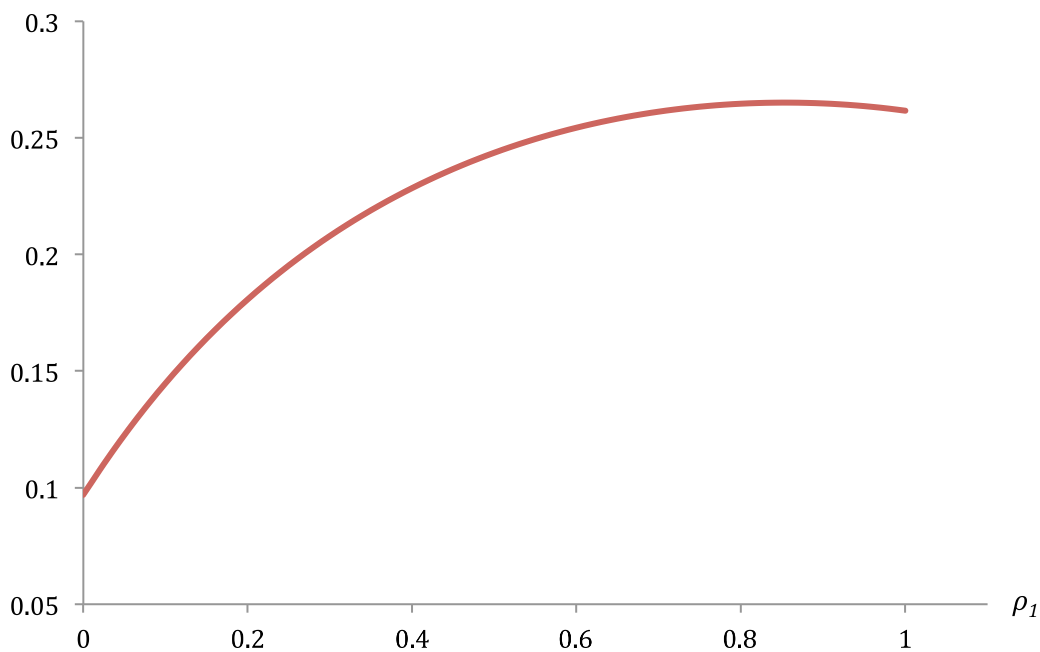

Figure 4. Profit Function for Informed Investor

This figure plots the profit for the informed volatility investor as a function of her strategy ρ_1. Parameter values are the same as described earlier, and $\alpha = 0.4, h_1 = 0.4$.

In solving the constrained maximization problem stated above, we obtain the equilibrium strategy as:

$$\rho_i^* = \frac{\alpha h_i C_j(\overline{\sigma})\sqrt{\left(C_i(\overline{\sigma})^2 - C_i\left(\underline{\sigma}\right)^2\right)} + (1-\alpha)h_i h_j\left[\sqrt{\left(C_i(\overline{\sigma})^2 - C_i\left(\underline{\sigma}\right)^2\right)}\left(C_j(\overline{\sigma}) + C_j\left(\underline{\sigma}\right)\right) - \sqrt{\left(C_j(\overline{\sigma})^2 - C_j\left(\underline{\sigma}\right)^2\right)}\left(C_i(\overline{\sigma}) + C_i\left(\underline{\sigma}\right)\right)\right]}{\alpha h_i C_j(\overline{\sigma})\sqrt{\left(C_i(\overline{\sigma})^2 - C_i\left(\underline{\sigma}\right)^2\right)} + \alpha h_j C_i(\overline{\sigma})\sqrt{\left(C_j(\overline{\sigma})^2 - C_j\left(\underline{\sigma}\right)^2\right)}} \tag{9}$$

See appendix for details.

Since the model does not permit negative probabilities (or short-selling one option contract to finance the other one), namely $\rho_i^* \leq 1\ \forall i$, the model can generate two types of equilibrium: Interior equilibrium or Boundary equilibrium.[8] An interior equilibrium ($0 < \rho_i^* < 1$) refers to the situation that is optimal for an informed trader to have a mixed strategy in selecting both contracts, whereas a boundary equilibrium ($\rho_i^* = 0\ or\ 1$) implies that, given the market parameters and environment, it is most profitable for the informed investor to trade only one option contract to exploit her informational advantage. Depending on the equilibrium type, the model will predict different results for relative information asymmetry, as well as for the pricing behavior of the options. Therefore, the discussions below are organized to show the conditions, if any, under which the equilibrium solution is an interior one, or when it is on the boundary.

Proposition 1.1:

An informed trader will **always** assign a positive probability for an OTM higher leveraged Contract-1.

$$\rho_1^* > 0 \tag{10}$$

Proof: see appendix

[8] This is very similar to the notion of ‘pooling equilibrium’ and ‘separating equilibrium’ in Easley, O’Hara, and Srinivas (1998). They consider two different markets, whereas I consider two option contracts with different characteristics, which allows for a direct comparison between the returns that the two option contracts may offer to the volatility traders.

This proposition implies that, in equilibrium, it is always optimal for the informed volatility trader to invest in the OTM contract to take advantage of the high leverage, regardless of the parameter value of $\alpha\ or\ h_1$. In fact, we can predict this result by learning from the graph of informed trader's profit function in Figure 4. Because of the higher marginal return associated with the more leveraged Contract-1, the positive slope on the left hand side of the graph indicates that the total profit for the informed trader will always increase with the positive probability in the OTM contract at the initial stage.

Proposition 1.2:

An informed trader will allocate all of her capital into the cheaper Contract-1 ($\rho_1^* = 1$) if and only if:

$$\frac{(1-\alpha)h_1}{\alpha} > \theta \tag{11}$$

$$where\ \theta = \frac{C_i(\overline{\sigma})\sqrt{C_j(\overline{\sigma})^2 - C_j(\underline{\sigma})^2}}{\left[\sqrt{C_i(\overline{\sigma})^2 - C_i(\underline{\sigma})^2}\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right) - \sqrt{C_j(\overline{\sigma})^2 - C_j(\underline{\sigma})^2}\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right]}$$

See appendix for proof.

This proposition shows that, given the properties of the two option contracts, it might be possible that, in equilibrium, the informed trader only trade the higher leveraged Contract-1. However, this could only happen if, in equilibrium, the ratio of liquidity hedger to informed trader is high enough in Contract-1 so that the informed trader is able to hide their orders in the market.[9] Here is a possible scenario where boundary equilibrium exists. Suppose that Contract-2

[9] The condition for the existence of boundary equilibrium strictly depends on the option contracts in the economy. Surprisingly, the right hand side of inequality (11) does not depend on the specific parameter values of the contracts

is ATM and Contract-1 is 15% OTM, then we have a threshold θ equal to 0.8867. We can interpret this as an informed investor will only invest all of her money in Contract-1with certainty if, ex-post, for every informed trade in Contract-1 there is approximately more than 0.8867 trade from uninformed traders. In this case, if we have an economy where there are 70% chance that a liquidity hedger is chosen ($\alpha = 0.3$) and a 40% chance that her capital is invested in OTM Contract-1 ($h_1 = 0.4$), then the ratio $\frac{(1-\alpha)h_i}{\alpha}$ equals 0.93, which is higher than the threshold, thereby sustaining a boundary equilibrium.

Theorem 1:

An interior equilibrium exists, that is an informed investor with private volatility information will split investment between two contracts, if and only if:

$$\frac{(1-\alpha)h_1}{\alpha} \leq \frac{C_1(\overline{\sigma})\sqrt{C_2(\overline{\sigma})^2 - C_2(\underline{\sigma})^2}}{\left[\sqrt{C_1(\overline{\sigma})^2 - C_1(\underline{\sigma})^2}\left(C_2(\overline{\sigma}) + C_2(\underline{\sigma})\right) - \sqrt{C_2(\overline{\sigma})^2 - C_2(\underline{\sigma})^2}\left(C_1(\overline{\sigma}) + C_1(\underline{\sigma})\right)\right]} \tag{12}$$

By combining proposition 2.1 and 2.2, we can obtain this theorem. It suggests that given the option prices in Time 1, if this condition on the relative player participation in the market is satisfied, it is optimal for the informed trader to invest in both option contracts at time 0 to maximize her profit. If not, according to Proposition 1.2, she will choose to invest all of her capital into the cheaper Contract-1 in order to take full advantage of the leverage. This theorem has a similar intuition to the 'pooling equilibrium' criteria between stock and option markets in Easley, O'Hara, and Srinivas (1998). Both models suggest that a relatively balanced liquidity

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if Black-Scholes values are used as an equilibrium value in Time 1. Rather, the threshold only depends on the moneyness of the OTM option contract, regardless of the specific stock or strike price. To illustrate some examples: (i) in the standard example: $S = k_2 = 30, k_1 = 35$, Contract-1 is 16.3% OTM, θ=0.8739; (ii) $S = k_2 = 100, k_1 = 115$, Contract-1 is 15% OTM, θ=0.8867; (iii) $S = k_2 = 50, k_1 = 57.5$, Contract-1 is 15% OTM, θ=0.8867.

between contracts (markets) is essential for an informed investor to trade in both securities. But this paper is able to provide a more direct interpretation of the liquidity ratio in terms of the actual option price and characteristics, both qualitatively and quantitatively.

Once the equilibrium solution for the informed investor's strategy is derived, we can infer their actions as the economic environment varies using some comparative statics analyses. It is well documented in existing literature that the market environment will change from time to time. For instance, the total number of informed traders relative to uninformed traders may increase[10] around days of critical importance such as event announcement days. Liquidity hedgers could also change their trading behaviors due to other exogenous factors. This model could shed light on how informed investors adjust their optimal strategy in equilibrium as the market structure changes. The following propositions come from key comparative statics that are analytically derived, which can help predict how informed volatility trader change her behavior when deciding how to strategize her investment between the two contracts. The primary results below are based on local derivative analysis, meaning that these propositions are only valid when considering the interior equilibrium; comments regarding boundary equilibrium will be made whenever necessary.

Proposition 1.3:

As the overall percentage of informed volatility investors increases, their optimal strategy in the higher leveraged contract falls.

$$\frac{d\rho_1^*}{d\alpha} < 0 \tag{13}$$

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[10] This is the same as saying that the probability of an informed trader being chosen has increased, so I will be using the two descriptions interchangeably. Similar arguments will also be applied for the probability of a liquidity hedger being selected.

Proposition 1.4:

As h_1 increases, i.e., the liquidity hedgers increase their probability for Contract-2, the informed trader's optimal strategy is also to increase their allocation in the higher leveraged Contract-1.

$$\frac{d\rho_1^*}{dh_1} > 0 \tag{14}$$

Intuitively, it will be more difficult for informed traders to hide their orders in the market as their population in the market rises. To reduce their presence in the eyes of market makers and thus maintain the leverage advantage of OTM Contract-1, informed investors must reduce their probabilityof total investment in Contract-1.

Proofs for both propositions are contained in the appendix. When the liquidity hedgers' trades move across to the cheaper contract, total liquidity in this contract increases. Because market makers observe this, an informed investor can thus increase their allocation in Contract-1 for higher profit without increasing market maker's information risk. In other words, it helps the informed investors to shift their investment into Contract-1 and improve their total profit without increasing the price. See the appendix for proofs on both propositions. Figure 5 presents an example of the optimal behavior of an informed volatility trader under a different combination of market conditions. As illustrated in Propositions 1.3 and 1.4, we see that an informed volatility trader will invest more aggressively in an OTM contract as the higher percentage of the liquidity hedger's transactions goes to the OTM contract or as the overall concentration of informed traders (α) decreases. The flat portion of this graph on the northeast corner represents all of the conditions under which boundary equilibrium is reached; in this example, threshold (θ) is 0.8379.

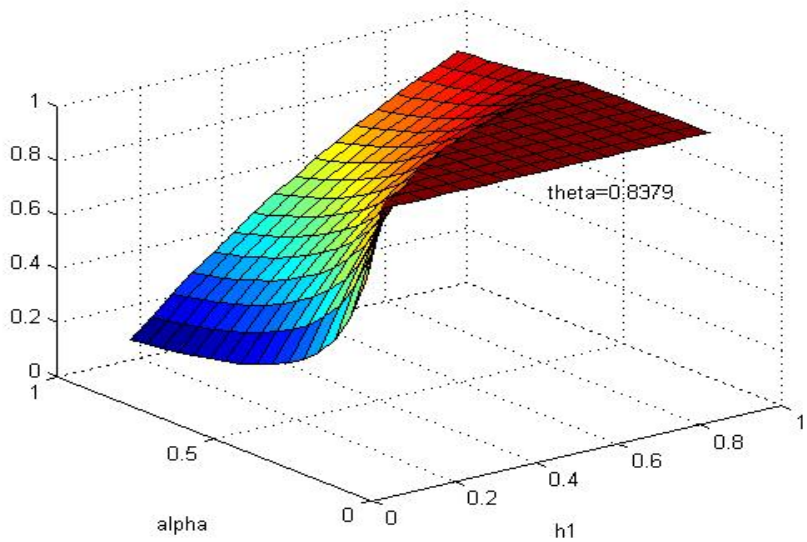

Figure 5. Optimal Strategy for Informed Volatility Investor

This figure plots an example of an informed volatility trader's optimal strategy under different combinations of α and h_1. The option contract used in this example is the standard one.

A critical objective of this model is to provide another framework that attempts to explain how option prices and the implied volatility behave under different structures of information asymmetry while incorporating the strategic behavior of an informed investor. Conventional wisdom may suggest that that informed traders should participate more in the OTM contract because of its higher implicit leverage. Interestingly enough, in a market such as the example in Figure 5, there are indeed many conditions that could support informed traders strictly preferring an OTM contract ($\rho_1 > 0.5$). However, it is imperative to be able to differentiate the notions of the informed trader's behavior and the level of information asymmetry. While the former characterizes the informed investor's preference over the two contracts (and her optimal strategy in investing them), the latter is of more importance to the market maker and pricing of options, which is determined by the relative participation between the informed trader and the liquidity hedger. One contributing feature of this paper is that the level of information asymmetry in each option contract is determined endogenously. As a result, the degree of information asymmetry in equilibrium could easily be different across the two contracts. If exogenous factors influence option prices such that there will be higher information asymmetry in the OTM contract than in the ATM contract, the difference in their prices, as well as the implied volatilities, could be even larger than what other models would have predicted by assuming a universal degree of adverse selection.

The Probability of Informed Trading (PIN) [Easley, Kiefer, and O'Hara (1996) and (1997)] is a conventional measure of information asymmetry used in many other papers. It attempts to capture the degree of information asymmetry in a particular financial market by calculating the probability that a trade in the market comes from an informed investor. In the context of this model, such probability for each option contract can be characterized as:

$$PIN_i^{Vol} = \frac{\alpha\rho_i^*}{\alpha\rho_i^* + 2(1-\alpha)h_i} \tag{15}$$

The numerator represents the probability that the market maker will receive an order from the informed trader for contract-i. The denominator is the total probability that the market maker will receive any order for contract-i in the next period.

Proposition 1.5:

The equilibrium level of information asymmetry about volatility is greater (weakly) in the OTM contract than the ATM contract. Namely:

$$PIN_1^{Vol} \geq PIN_2^{Vol} \tag{16}$$

This proposition shows that, under equilibrium, the best action by an informed volatility trader and market maker will always result in a higher volatility information risk in an OTM contract. The proof of Proposition 1.4 is included in the appendix. The process of this proof simply shows that as long as the OTM contract provides higher implicit leverage in Time 1, then the ATM contract has higher information risk than the OTM contract cannot be sustained under an interior equilibrium. Intuitively, this is because it is always in the best interest of the informed trader to increase the investment in the OTM contract as long as it provides a higher "bang-per-buck" than the ATM option. When we have a case where the information asymmetry is higher in the ATM contract, the market maker will adjust the ATM option price relatively higher than the OTM contract, meaning that Contract-1 has an even greater leverage advantage in Time 0 than the perfect information case. This incentivizes informed investors to continue investing more in Contract-1, which means that the previous case cannot be in equilibrium.

Table 1 provides a numerical example of Proposition 1.4. We see that the information asymmetry is predicted to be higher in OTM Contract-1 than in Contract-2 as a consequence of the equilibrium behavior from the informed investors for all combinations of parameters. This result is insensitive to the type of equilibrium because, in the case of boundary equilibrium, all informed traders would place their orders in Contract-1, which simply leads to zero asymmetric information for Contract-2.

Table 1. Simulated Levels of Information Asymmetry (PIN) under Equilibrium

Each pair of values represents the different level of asymmetric information (measured by PIN) for the two option contracts at equilibrium for a selected range of the market environment. The following parameters are used for options C-1 and C-2 as stated earlier: $\overline{\sigma} = 0.35$; $\underline{\sigma} = 0.25$, $k_1 = 35$; $k_2 = 30$; $S = 30$; $T = 45$; $r = 0.03$

Total % of informed (a1)	Hedger's Strategy in Contract-1 (h1)																	
	0.1		0.2		0.3		0.4		0.5		0.6		0.7		0.8		0.9	
	C-1	C-2	C-1	C-2	C-1	C-2	C-1	C-2	C-1	C-2	C-1	C-2	C-1	C-2	C-1	C-2	C-1	C-2
0.1	0.36	0.00	0.22	0.00	0.16	0.00	0.12	0.00	0.10	0.00	0.08	0.00	0.07	0.00	0.06	0.00	0.06	0.00
0.15	0.39	0.03	0.31	0.00	0.23	0.00	0.18	0.00	0.15	0.00	0.13	0.00	0.11	0.00	0.10	0.00	0.09	0.00
0.2	0.41	0.06	0.38	0.01	0.29	0.00	0.24	0.00	0.20	0.00	0.17	0.00	0.15	0.00	0.14	0.00	0.12	0.00
0.25	0.43	0.09	0.40	0.04	0.36	0.00	0.29	0.00	0.25	0.00	0.22	0.00	0.19	0.00	0.17	0.00	0.16	0.00
0.3	0.45	0.13	0.42	0.08	0.39	0.03	0.35	0.00	0.30	0.00	0.26	0.00	0.23	0.00	0.21	0.00	0.19	0.00
0.35	0.47	0.17	0.44	0.12	0.42	0.07	0.39	0.03	0.35	0.00	0.31	0.00	0.28	0.00	0.25	0.00	0.23	0.00
0.4	0.49	0.21	0.47	0.17	0.44	0.12	0.42	0.08	0.39	0.03	0.36	0.00	0.32	0.00	0.29	0.00	0.27	0.00
0.45	0.52	0.25	0.49	0.21	0.47	0.17	0.45	0.13	0.42	0.08	0.40	0.04	0.37	0.00	0.34	0.00	0.31	0.00
0.5	0.54	0.30	0.52	0.26	0.50	0.22	0.48	0.18	0.45	0.14	0.43	0.10	0.41	0.06	0.38	0.02	0.36	0.00
0.55	0.57	0.35	0.55	0.31	0.53	0.28	0.51	0.24	0.49	0.21	0.47	0.17	0.45	0.13	0.42	0.09	0.40	0.05
0.6	0.61	0.40	0.59	0.37	0.57	0.34	0.55	0.31	0.53	0.27	0.51	0.24	0.49	0.21	0.47	0.17	0.45	0.13
0.65	0.64	0.45	0.62	0.43	0.61	0.40	0.59	0.37	0.57	0.34	0.55	0.31	0.54	0.28	0.52	0.25	0.50	0.22
0.7	0.68	0.52	0.66	0.49	0.65	0.47	0.63	0.44	0.62	0.42	0.60	0.39	0.59	0.37	0.57	0.34	0.55	0.32
0.75	0.72	0.58	0.71	0.56	0.69	0.54	0.68	0.52	0.67	0.50	0.65	0.48	0.64	0.46	0.63	0.44	0.61	0.41
0.8	0.76	0.65	0.75	0.63	0.74	0.62	0.73	0.60	0.72	0.59	0.71	0.57	0.70	0.55	0.69	0.53	0.68	0.52
0.85	0.81	0.73	0.81	0.71	0.80	0.70	0.79	0.69	0.78	0.68	0.77	0.67	0.76	0.65	0.76	0.64	0.75	0.63
0.9	0.87	0.81	0.86	0.80	0.86	0.79	0.85	0.79	0.85	0.78	0.84	0.77	0.84	0.76	0.83	0.75	0.82	0.74

Proposition 1.4 also has profound empirical implications on a variety of topics. It is consistent with much of the empirical evidence regarding the informational content in OTM options. I show that information asymmetry is indeed greater in OTM options than in ATM options as an equilibrium outcome. For example, Chakraverty, Gulen and Mayhew find that the information share is the greatest for OTM options using econometric estimations; and Pan and Poteshman (2006) find that information embedded in more OTM option contracts has greater predictive power than ATM options. In the following section, I discuss how this proposition,

and its extensions, can help explain the spread structure and the implied volatility skew in option markets.

2.4.2 *Option Price in Equilibrium*

In this model, the equilibrium price at Time 0 is the market maker's expected value of the contract, given the optimal strategy of the informed traders. Specifically, if we insert equilibrium ρ_i^* into the expression for the ask-price, we can obtain the equilibrium ask-price for contract-i as:

$$C_0(k_i) = A_i = \frac{\alpha\rho_i^* + (1-\alpha)h_i}{\alpha\rho_i^* + 2(1-\alpha)h_i} * C_i(\overline{\sigma}) + \frac{(1-\alpha)h_i}{\alpha\rho_i^* + 2(1-\alpha)h_i} * C_i(\underline{\sigma}) \quad (17)$$

This can be interpreted as a weighted average between the high-volatility state price and low-volatility state price, where the weights are the equilibrium conditional probabilities with respect to each state from the perspective of the market makers. Similar to other asymmetric information models in existing literature, whenever there is an exogenous shock to the market environment, i.e., a change in the parameter value of α and h_i, market makers will update their beliefs according to these exogenous factors and thus cause price movement. One of the key features this model is attempts to characterize is the presence of a secondary effect on the option price following the initial exogenous shock; the informed trader's optimal strategy will endogenously guide her to also adjust investment behavior, thus further affecting the market maker's belief and information asymmetry and moving the equilibrium price accordingly. Simply put, because ρ_i^* is also a function of α and h_i in equilibrium, a change in either of the parameters will change ρ_i^*, which further affects the option price of each contract.

One expects the ask-price to increase with the level of informed traders but decrease with the number of liquidity hedgers in the market according to existing theories. However, since the informed traders' choice also depends on those two factors, the secondary effect has the potential to move the price in the opposite direction. For instance, because the informed trader's optimal strategy depends on the liquidity hedger's strategy in the contracts, a shift of liquidity hedger strategy from Contract-1 to Contract-2 will initially increase the probability of liquidity traders in Contract-2 and decrease its price according to equation (17). But this will be followed by the adjustments of the informed investor's investment strategy; by Proposition 1.4, the informed investor should also increase their strategy in Contract-2, which may offset some impacts caused by the initial change of liquidity hedgers.

Proposition 2.1:

The ask-price of both option contracts will increase with the overall share of informed traders in the market:

$$\frac{dA_1^*}{d\alpha} > 0 \tag{18}$$

$$\frac{dA_2^*}{d\alpha} > 0 \tag{19}$$

These results are not trivial, specifically because of the secondary effect mentioned earlier. While the overall level of information asymmetry increases in this market, the prices for both options will initially be subject to upward pressure. However, as the market price begins to rise, the relative leverage advantage of Contract-1 will fall, causing some informed investment to shift to Contract-2 to seek higher profit margins. This effect will further push up the price of option Contract-2. However, the key point here is that it will also reduce the information asymmetry in Contract-1, which affects the price in the opposite direction as the initial pressure. This proposition demonstrates that the initial upward pressure will always outweigh the secondary leverage effect in Contract-1. Proof of this proposition can also be found in the appendix. As for a boundary solution, the effect of changes in α on option prices is relatively easy to decipher. An increase in the overall level of informed traders will only increase the information asymmetry in Contract-1, but not Contract-2 because there is no informed investor present in Contract-2 in a boundary equilibrium.

The impact of an exogenous movement in the liquidity hedger's preference on the option price is more complex to analyse. For instance, if the liquidity hedger has a higher probability of investing her capital into the OTM Contract-1, i.e., h_1 increases (h_2 decreases), the liquidity in Contract-1 (Contract-2) will initially improve (worsen), thus decreasing (increasing) the option price. However, an increase in h_1 will also increase the optimal strategy of an informed trader in Contract-1 (ρ_1^*), in the equilibrium, which will increase (decrease) the option price for Contract-1 (Contract-2) as the degree of adverse selection problem in each contract changes. In this case, whenever the change in the market environment results from the liquidity hedger's behavior, the secondary "leverage" effect will push the option price in the opposite direction of the initial price movement. Figures 6 and 7 illustrate a numerical example for both option contracts that the information asymmetry, thus the price, will decrease with the probability of the liquidity hedger's investment in the OTM Contract-1 (h_1).

A_1:

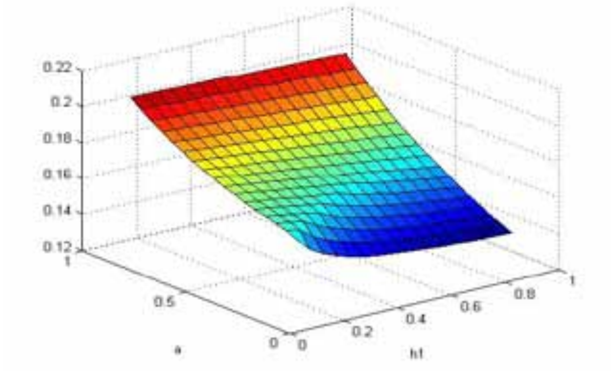

Figure 6. Equilibrium Ask-Price of Contract-1

This graph demonstrates the ask-price for *Contract-1*. The vertical z-axis is the ask-price. Left horizontal axis is the value of α; right horizontal axis is the value of h_1.

A_2:

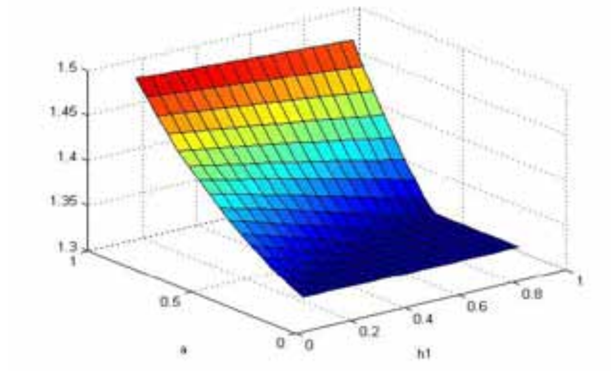

Figure 7. Equilibrium Ask-Price of Contract-2

This graph demonstrates the ask-price for *Contract-2*. The vertical z-axis is the ask-price. Left horizontal axis is the value of α; right horizontal axis is the value of h_1.

Furthermore, Figures 6 and 7 demonstrate Proposition 2.1; as the value of α rises, the equilibrium information asymmetry increases in both of the markets, pushing up the market maker's conditional expected value and the price of both options when seeing a BUY order, despite the fact that increasing α would alternate an informed trader's strategy in this market. Notice that there is also no effect on the ask-price of Contract-2 as h_1 changes for small values of α; this is because under these market conditions, we are at the boundary solution and there is no informed trader's presence in the ATM contract. Any marginal changes in h_1 that do not change an informed trader's optimal strategy will not change the ask-price of Contract-2.

However, one seemingly counter-intuitive result needs to be emphasized: this model predicts that the ask-prices for both Contract-1 and Contract-2 decrease with h_1. As the probability of liquidity hedger's investment increases (decreases) in Contract-1 (Contract-2), this initially decreases (increases) the degree information asymmetry in the respective market, which helps to push the ask-price down (up). At the same time, Proposition 1.4 shows that an informed trader's strategy moves toward (away from) the more leveraged OTM Contract-1 (less leveraged

Contract-2) as a result of the secondary 'leverage' effect, which increases (decreases) the ask-price. It seems that the prices for two contracts should end up moving in opposite directions after the information structure is redefined in the market. But the graphs demonstrate a different story. The fact that the ask-price for Contract-1 decreases implies that the initial effect in this contract is greater than the secondary effect that comes from the informed trader's adjustment. To the contrary, the fact that we see the ask-price of Contract-2 also fall indicates that the relative magnitude of the opposite secondary "leverage" effect is greater for Contract-2. More importantly, this implies that the information asymmetry under the new equilibrium will decrease in both contracts in this example.

2.4.3 Implied Volatility and Shape of the Skew (Smile)

There have been a number of previous models that try to explain the empirical phenomenon of an implied volatility smile in option prices. Among them, Cherian and Jarrow (1998) are the first to incorporate information asymmetry about the future volatility of an asset into the discussion of option pricing. While they clearly show that an implied volatility smile can be generated because the equilibrium option prices are expectations of Black-Sholes prices due to the asymmetric information between market makers and informed traders, they implicitly assume that informed volatility traders treat each option contract the same. However, as demonstrated in previous sections, the level of information asymmetry can easily be different between option contracts as a result of implicit leverage and liquidity. In fact, Proposition 1.4 shows that information risk in an OTM contract is always no less than in an ATM contract. In this section, I provide a detailed discussion on the impact of heterogeneity in information asymmetry (a new feature of this model) on the relative expensiveness (measured by implied volatility) of OTM and ATM contracts. In particular, I elucidate why an informed volatility trader's behavior will affect the skewness (slope) of the volatility smile. Furthermore, the manner in which the dispersion in information asymmetry between option contracts varies from time to time will also undoubtedly affect the shape of the volatility smile over time. I demonstrate how the changes in different model parameters will change the equilibrium outcome, allowing us to better understand the dynamic evolution of the IV skew as well as the cross-section variations.

Recall that in equilibrium, the ask-price of a call option can be expressed as the expected value of the option based on the conditional probability of each volatility state:

$$A_i = pr_i(\overline{\sigma}|BUY) * C_i(\overline{\sigma}, k_i) + pr_i(\underline{\sigma}|BUY) * C_i(\underline{\sigma}, k_i) \; for\; i = 1{,}2 \tag{20}$$

$$where\; pr_i(\overline{\sigma}|BUY) = \frac{\alpha\rho_i + (1-\alpha)h_i}{\alpha\rho_i + 2(1-\alpha)h_i}$$

Now, let us define the Black-Scholes implied volatility based on the equilibrium ask-price in the market, as:

$$A_i = E_i[C(\sigma, k_i)|BUY] = BS(\boldsymbol{a_i}, k_i) \tag{21}$$

$$where\; \boldsymbol{a_i}\; is\; the\; implied\; volatility\; in\; equilibrium\; option\; price$$

Although the skewness of implied volatility (IV) is typically described as the slope of the smile on either side of the ATM strike price; because there are only two option contracts in this model, IV skew is defined as the vertical distance between the implied volatility of Contract-1 and Contract-2 as shown in Figure 8:

$$skew = a_1 - a_2 \tag{22}$$

$$where\; a_{1,2}\; is\; the\; implied\; ask\; volatility\; of\; each\; contract$$

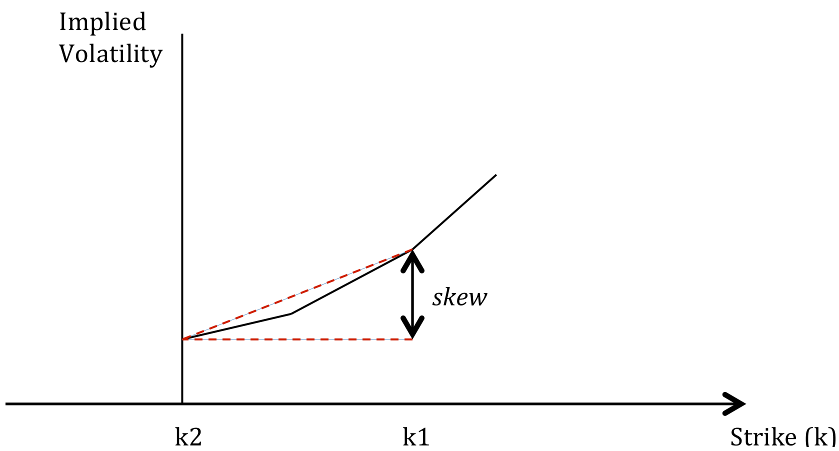

Figure 8. Characterize Implied Volatility Skew

This Figure illustrates how Implied Volatility Skew is defined and measured in this model. In this most simplistic economy, IV Skew is measured by the vertical distance in terms of B-S implied volatility between Contract 1 and 2.

From equation (20) through equation (22), it is critical to notice that there are two aspects that determine the level of implied volatility in the model. First, the range of volatility uncertainty, which is governed by the volatility parameters ($\{\overline{\sigma}, \underline{\sigma}\}$) in the model, will clearly affect a_i. Moreover, the conditional probability of a high volatility state given a BUY order for the contract has arrived ($pr_i(\overline{\sigma}|BUY)$), is an equally important term that decides the level of implied volatility of each option contract. It can be easily shown that the relationship between these probabilities associated with either contract is the same as the relationship between their respective levels of Probability of Informed Trading (PIN). For example, if $pr_1(\overline{\sigma}|BUY) = pr_2(\overline{\sigma}|BUY)$, which is the same as $PIN_1 = PIN_2$[11], then the information asymmetry in both contracts is identical. In this case, this model will generate an implied volatility skew that can be predicted by a reduced version of the model in Cherian and Jarrow (1998). However, as shown earlier, the likely optimal strategy for an informed volatility trader in equilibrium would lead to a higher level of information asymmetry in volatility for the OTM contract. When $PIN_1 > PIN_2$ (equivalent to $pr_1(\overline{\sigma}|BUY) > pr_2(\overline{\sigma}|BUY)$), the model produces a skew level that is greater than the case of homogeneous information asymmetry. It is important to

[11] This relationship between PIN_i and $pr_i(\overline{\sigma}|BUY)$ can be proved using the property that probabilities add up to one. An example of this proof can be found in appendix.

note that the more the probability of Contract-1 exceeds the probability of Contract-2, the greater the implied volatility skew. Hence given the same volatility parameters, the degree of volatility skewness is governed by the size of the information risk differentials.[12] Figure 9 demonstrates the effect of heterogeneous information risk on the IV smile. To simulate Figure 9, it is assumed that the informed volatility trader's strategy is determined based on the parameters in previous examples, and the market condition is such that α=0.4 and h_1=0.4. The blue volatility smile is calculated using the priors of ATM Contract-1 ($pr_2(\overline{\sigma}|BUY)$) for all strikes. The red dotted line represents the shape of the IV smile obtained by using the priors of ATM Contract-1 ($pr_2(\overline{\sigma}|BUY)$) for strike prices less than and equal to 30, but using the priors of OTM Contract-1 ($pr_1(\overline{\sigma}|BUY)$) for OTM strike prices greater than 30. It is clear in this hypothetical simulation that under heterogeneous information asymmetry, because there is higher information risk in OTM contracts, the OTM options will be priced much higher than in the case of identical information asymmetry, resulting in a greater implied volatility skew.

[12] It should be noted that effect of information risk differentials on implied volatility skew can also be partly attributed to the non-linearity of the B-S formula in the strike price.

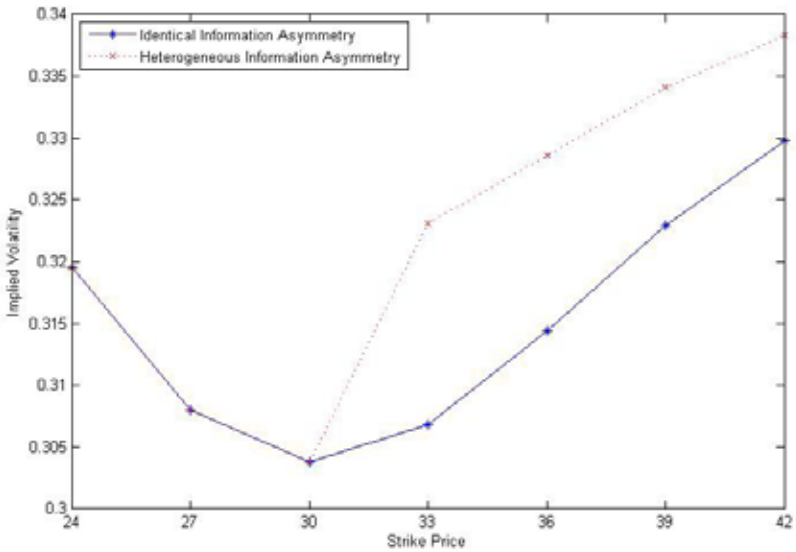

Figure 9. Implied Volatility Smile under Heterogeneous Information Asymmetry

This figure demonstrates an example of different shapes of implied volatility smiles under two information structures. To generate this graph, I first obtain the information risk in probabilities of ATM and OTM contracts using the parameters in the standard example. Then, I use only the information risk in the ATM contract to simulate option IV in the blue line; I plot the red line with the ATM information risk for strikes less and equal to 30 and use the OTM information risk to simulate option prices for strikes greater than 30.

Because the value of p_i directly relates to the relative information asymmetry, which depends on informed trader's action predicted by the model, I will carefully provide intuitions as well as examples to illustrate the new predictions of this model. In particular, I am going to discuss how the evolution of the information structure and the liquidity hedger's preference lead to different equilibrium actions and prices that have a significant impact on the shape of the implied volatility skew. For instance, if the equilibrium results in a higher fraction of informed investors in the total trading volume of the OTM option contract than in the ATM contract, the difference in implied volatility between these two contracts will be greater than if the concentration of informed investors is the same for both contracts.

Proposition 3.1:

Given the same volatility uncertainty, a higher degree of heterogeneity in information asymmetry between OTM and ATM contracts will lead to a greater implied volatility skew.

The intuition behind this proposition is quite straightforward. Because implied volatility is an increasing function in option price, other things being equal, the market with greater difference in information risk between contracts will price an OTM contract higher in equilibrium, resulting in a greater IV skew. Figure 10 is a simulated example of implied volatility skew as defined in equation (22) and Figure 8, where we can see that the skewness is positive for all parameter values. More importantly, if we compare Figure 10 against the adjacent Figure 11, where the difference in information asymmetry ($PIN_1 - PIN_2$) is plotted under the same scale of x and y axis, we clearly observe the linkage between the slope of the implied volatility skew and the degree of heterogeneity in information asymmetry as suggested in Proposition 3.1.

The implication of this proposition also provides an alternative explanation to some existing empirical findings regarding the correlation between bid-ask spreads and the slope of the IV smile. In addition to the positive relationships between the IV skew and the difference in information asymmetry across strike prices, a higher information risk will also lead to a greater spread for an OTM contract (details of the spread discussion can be found in the next section). Therefore, the difference in information asymmetry between option moneyness serves as the linkage between the IV skew options and the difference in their spreads. This prediction is consistent with the evidence in Peña, Rubio and Serna (1999), in which they document a statistically significant relationship between the difference in option spreads and the slope of the

IV smile.[13]

[13] However, Peña, Rubio and Serna (1999) have a different hypothesis for this relationship in their paper. They conjecture that the larger spread represent higher transaction cost in an OTM contract, which leads to greater implied volatility.

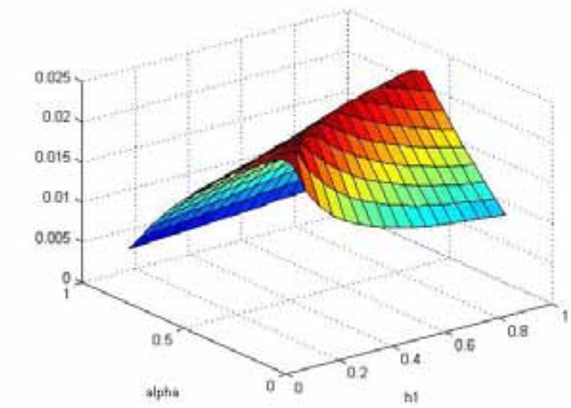

Figure 10. Impact of Market Condition on IV Skew

The skewness, or the slope of the implied volatility smile, is plotted against different parameter values; standard parameter values.

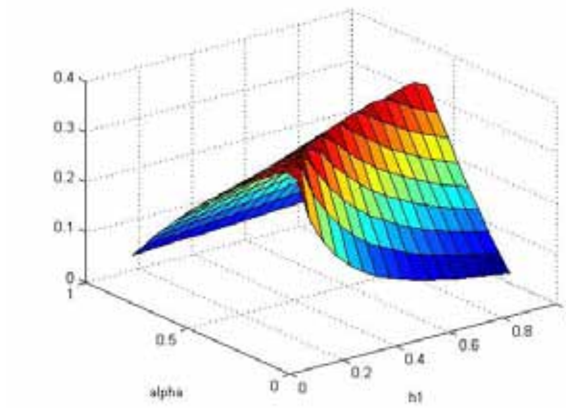

Figure 11. Impact of Market Condition on Difference in Information Asymmetry

The difference in information risk (PIN_1 -PIN_2) is plotted against different parameter values.

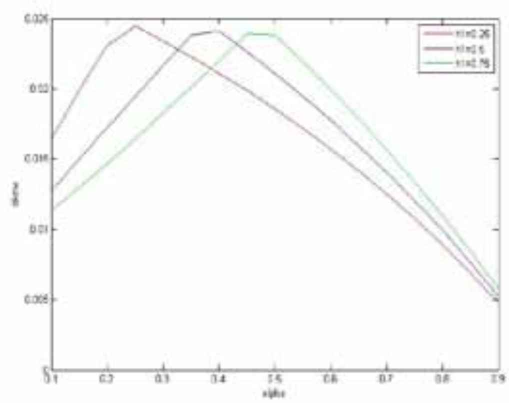

Figure 12. Impact of Overall Concentration of Informed Trader on IV Skew

The level of skew is plotted against values of α for three different values of h_1.

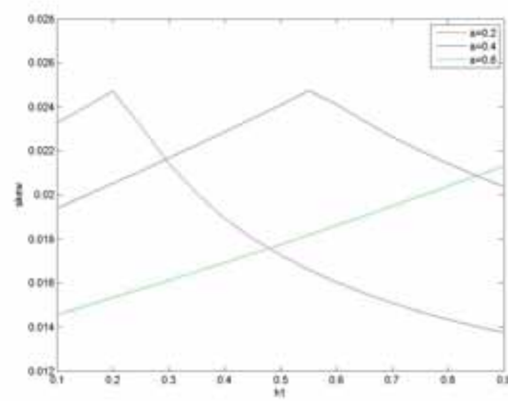

Figure 13. Impact of Behavior of Liquidity Hedger on IV Skew

The level of skew is plotted against values of h_1 for three different values of α.

In order to explore the relationship between the implied volatility skew and the market environment in further detail, let us look at Figures 12 and 13, where the two-dimensional snapshots of the skewness graph in Figure 10 are presented. First, it should be noted that we observe a kink in Figure 12 because all informed investors' orders might be traded in the OTM contract for some parameter values sustaining a boundary equilibrium. In those circumstances when α increases, the optimal strategy in the OTM contract for the informed trader, ρ_1^*, does not change initially (stays at 1); this will only increase the information asymmetry and the price of option Contract-1 while the price of Contract-2 remains unaffected. So a higher α widens the discrepancy of information asymmetry between contracts and increases the slope of the smile. However, as α rises further, we move to interior equilibrium at which informed traders will participate in both contracts. In this case, the more overall level of informed traders, the higher probability of investing in Contract-2 in equilibrium. This secondary effect will decrease (increase) the information asymmetry in Contract-1 (Contract-2), which reduces the heterogeneity of the implied volatility between the two contracts. Therefore, the volatility skew decreases with α for interior equilibrium solutions. One implication of this result is that the model predicts a decrease in the implied volatility skew during the days leading up to an important announcement day provided the market is in an interior equilibrium, as the overall level of informed traders tends to increase before announcement periods. As for the relationship between skewness and the liquidity hedger's behavior, previous results indicate that when a liquidity investor's investment preference shifts to OTM Contract-1, the information asymmetry will fall in both contracts. Therefore, the relative magnitude of change will have a consequential impact on the shape of the smile. Figure 13 shows that whenever the equilibrium solution is NOT at the boundary (i.e., $0 < \rho_1^* < 1$), the skewness of the option market is increasing with the hedger's probability of choosing OTM Contract-1 while both options' prices are falling. However, for low values of α, the slope of the smile might initially increase as both prices fall. Yet as soon as the equilibrium solution moves to the boundary where the informed investor only trades in Contract-1, any further increase in h_1 will only reduce the information asymmetry—and thus the price—of Contract-1 while increasing both the information asymmetry and the price of Contract-2; thus, the volatility skew will begin to decrease. Overall, any changes in exogenous market conditions that lead to variations in Probability of Information Trading in each moneyness contract could result in different level of skewness. For example, consider the empirical implication of Proposition 3.1 on the cross-section features of the IV skew. If there are two stocks with very similar prices and volatility uncertainty (or two currency pairs with similar exchange rates and volatility in the case of foreign exchange markets), but the options of one asset attracts a significantly higher population of informed volatility investors (relative to liquidity hedgers) than the options of the other asset. The level of heterogeneity between information risks across the strike for the two corresponding option series would then be different, which means that one series will have a higher skew than the other.

As discussed earlier, equation (20) also suggests that volatility uncertainty in the market (governed by parameter $\{\overline{\sigma}, \underline{\sigma}\}$) plays an important role in determining the exact price, thus the level of implied volatility. In fact, changes in volatility parameters will also affect the IV skew in two steps. Firstly, there is the direct price effect, as the values of $C_i(\overline{\sigma}, k_i)$ and $C_i(\underline{\sigma}, k_i)$ will be affected. Secondly, this will also affect the equilibrium prices, thus the IV skew, through the information structure of the option series. More importantly, the magnitude of the latter depends crucially on the conditions of each market. Figure 14 illustrates the effect of change in volatility uncertainty on the IV skew with a numerical example. As the range of uncertainty (measured by $\overline{\sigma} - \underline{\sigma}$) increases, we can see that the level of IV skew increases in the market represented by the red line, where α=0.1, h_1=0.8, and the average PIN difference between OTM and ATM contracts is 0.0649. In contrast in the market condition represented by the blue line, where α=0.6 and h_1=0.8, the average degree of heterogeneity in information asymmetry is at a much higher level of 0.2618; we see that an increase in the range of volatility uncertainty will actually decrease the level of IV skew. From Figure 14, we see that due to the unique feature of this model, the implied volatility skew is jointly determined by volatility parameters and the structure of information risk. Not only will the equilibrium information structure across strikes result in a higher absolute level of IV skew, but the information risk differentials between the option contracts also has the potential to influence the degree to which a change in volatility uncertainty can affect implied volatility skew. This empirical implication will be further examined in detail in the following chapter.

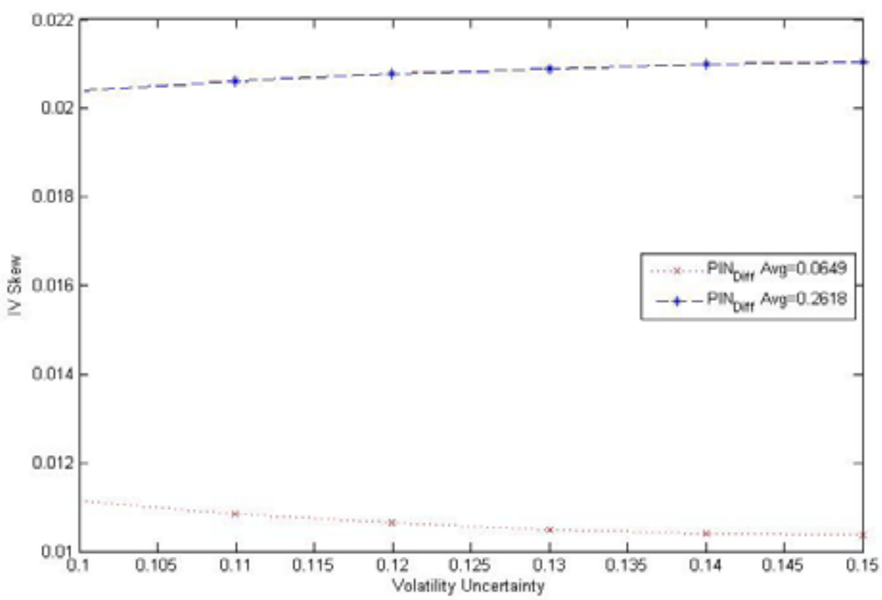

Figure 14. Impact of Volatility Uncertainty on IV Skew

This figure shows how changes in volatility uncertainty (measured by $\overline{\sigma} - \underline{\sigma}$) affect IV skew under different market conditions. The blue line represents a case that the average difference in Probability of Information Trading between OTM and ATM contracts is 0.2618; the red line represents the case that this difference is 0.0649.

2.4.4 Bid- Ask Spread

Information risk has long been regarded as an important factor in determining the bid-ask spread in financial markets. A number of theoretical papers, beginning with Glosten and

Milgrom (1985) and Easley and O'Hara (1987), model the relationship between information asymmetry and bid-ask spread equity markets. Extensions of these structural models are also used to address specific issues in the financial markets. For example, Cherian and Jarrow (1998) develop an asymmetric information model to better understand the relationship between information risk and option market spreads. In this model, I attempt to contribute to this literature by theoretically illustrating the role of volatility information trading in the spread structure of option markets. Specifically, I will demonstrate how the heterogeneity in the information asymmetry across option contracts affects the relationship between their spreads. We know from standard market microstructure theories that factors such as information risk and market liquidity tend to influence the bid-ask spreads of financial securities. For financial markets such as option markets, in which you have more than one security being traded, it would be interesting to investigate if the spreads for different option contracts are inter-connected, and moreover, how does such connection reacts to parameter changes in this model.

Results from previous sections have shown that in equilibrium, relative information asymmetry about volatility can be considerably different between the contracts due to the strategic interaction between the informed trader and the market maker. According to Proposition 1.4, this model predicts that an ATM contract always has information risk that is less or equal to the risk of an OTM contract. Because volatility uncertainty is a risk that cannot be easily hedged away by the market maker, the information structure within the same option series should therefore determine the spread structure of these options. It would be natural to expect that the spread structure is consistent with previous findings: a bid-ask spread for an OTM option should be higher than an ATM option due to the greater information risk for the market maker; furthermore, the spread of Contract-i should increase with α and decrease with h_i. The section below attempts to confirm these conjectures. Finally, I attempt to shed light on the relationship between relative information asymmetry and the spread difference across strikes, thus answering some practical questions such as whether an increase in overall level of informed volatility traders will increase or reduce the spread difference between the contracts.

To establish the bid-ask spread in this market, I am going to first derive the bid-price as the conditional expectation of call prices in a very similar fashion as the ask-price, but the condition is that given the market maker receives a SELL order:

$$B_i = E[C(\sigma, k_i)|SELL]$$

$$= pr(\overline{\sigma}|SELL) * C_1(\overline{\sigma}, k_i) + pr(\underline{\sigma}|SELL) * C_1(\underline{\sigma}, k_i)$$

$$= \frac{(1-\alpha)h_i}{\alpha\rho_i + 2(1-\alpha)h_i} * C_1(\overline{\sigma}, k_i) + \frac{\alpha\rho_i + (1-\alpha)h_i}{\alpha\rho_i + 2(1-\alpha)h_i} * C_1(\underline{\sigma}, k_i)$$

The informed investor attempts to maximize total profit by selling options in a low volatility state, $\sigma = \underline{\sigma}$:

$$Max_{\rho_i} \ \frac{w\rho_i}{B_i}\left(B_i - C_i(\underline{\sigma})\right) + \frac{w\rho_j}{B_j}\left(B_j - C_j(\underline{\sigma})\right)$$

$$s.t. \ \ \rho_i + \rho_j = 1$$

In equilibrium, the optimal strategy and bid-prices are shown below:

$$\rho_i^{S^*} = \frac{\alpha h_i C_j(\underline{\sigma})\sqrt{\left(C_i(\overline{\sigma})^2 - C_i(\underline{\sigma})^2\right)} + (1-\alpha)h_i h_j\left[\sqrt{\left(C_i(\overline{\sigma})^2 - C_i(\underline{\sigma})^2\right)}\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right) - \sqrt{\left(C_j(\overline{\sigma})^2 - C_j(\underline{\sigma})^2\right)}\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right]}{\alpha h_i C_j(\underline{\sigma})\sqrt{\left(C_i(\overline{\sigma})^2 - C_i(\underline{\sigma})^2\right)} + \alpha h_j C_i(\underline{\sigma})\sqrt{\left(C_j(\overline{\sigma})^2 - C_j(\underline{\sigma})^2\right)}} \quad (23)$$

$$B_i = \frac{(1-\alpha)h_i}{\alpha\rho_i^{S^*} + 2(1-\alpha)h_i} * C_i(\overline{\sigma}) + \frac{\alpha\rho_i^{S^*} + (1-\alpha)h_i}{\alpha\rho_i^{S^*} + 2(1-\alpha)h_i} * C_i(\underline{\sigma}) \quad (24)$$

Therefore, the equilibrium spread for each option contract is characterized as below:

$$Spread_i = A_i^* - B_i^* \tag{25}$$

$$= \left(\frac{\alpha \rho_i^{B^*} + (1-\alpha)h_i}{\alpha \rho_i^{B^*} + 2(1-\alpha)h_i} * C_i(\overline{\sigma}) + \frac{(1-\alpha)h_i}{\alpha \rho_i^{B^*} + 2(1-\alpha)h_i} * C_i(\underline{\sigma}) \right)$$

$$- \left(\frac{(1-\alpha)h_i}{\alpha \rho_i^{S^*} + 2(1-\alpha)h_i} * C_i(\overline{\sigma}) + \frac{\alpha \rho_i^{S^*} + (1-\alpha)h_i}{\alpha \rho_i^{S^*} + 2(1-\alpha)h_i} * C_i(\underline{\sigma}) \right)$$

Figures 15 and 16 depict the spread level of each individual contract for a range of parameter values. The conventional results from existing theories still hold, but with minor differences. Namely, the model projects that the spreads for both options will increase with α, which increases information asymmetry, but this only happens for the interior equilibrium. The spread remains unchanged for Contract-2 in boundary equilibrium because the information risk, being the only factor that affects the spread in this model, does not change in these cases. Additionally, both spreads decrease with h_i, which measures the concentration of informed traders in that contract. As long as the economy is in interior equilibrium, this prediction is again consistent with previous results.

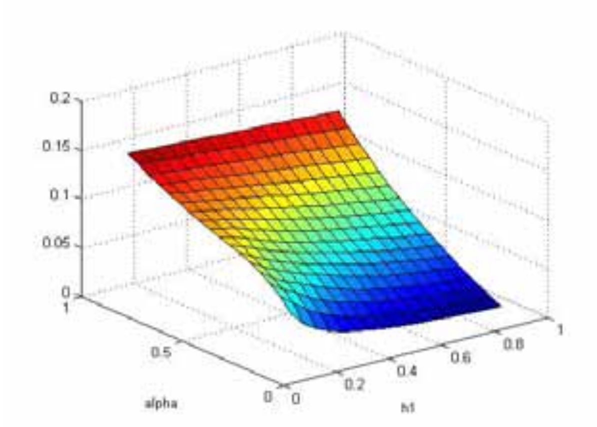

Figure 15. Equilibrium Spread of Contract-1

This figure shows the predicted spread for *Contract-1*. The vertical z-axis is the ask-price. Left horizontal axis is the value of α; right horizontal axis is the value of h_1.

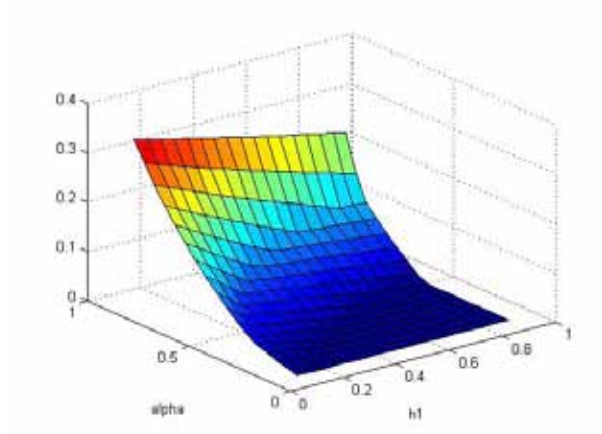

Figure 16. Equilibrium Spread of Contract-2

This figure shows the predicted spread for *Contract-2*. The vertical z-axis is the ask-price. Left horizontal axis is the value of α; right horizontal axis is the value of h_1.

I next consider the relationship between the spreads in ATM and OTM options. Because the absolute values of option prices usually vary significantly across contracts with different strikes. I use two different variables—*percentage spread* and *implied volatility spread*— to show a meaningful analogy across contracts.

Define percentage spread as:

$$Spread_i\% = \frac{A_i^* - B_i^*}{\frac{1}{2}(A_i^* + B_i^*)} \quad (26)$$

Define implied volatility spread as:

$$Implied\ Volatiliy\ Spread_i = a_i - b_i \quad (27)$$

where a_i and $b_i$is the implied volatility derived from Ask and Bid price, respectively.

First of all, Figures 17 and 18 show that the spread for OTM call options is always greater than the contract with a smaller strike price in equilibrium, regardless of which measurement is used. This is consistent with the empirical observation that the spread of OTM contracts is usually much greater than the spread of ATM contracts; it is a direct consequence of the prediction that OTM contracts have greater information asymmetry in equilibrium. Secondly, Figure 17 shows that as the overall probability of informed traders in this economy increases, the disparity between the spread enhances. Whereas such difference in percentage spreads between contracts shrinks as the preference of the hedger shifts into Contract-1. But what is intriguing is that if we look at the difference in implied volatility spread in Figure 18, it displays a completely different pattern from the percentage spread difference. It initially increases with the total percentage of the informed investor in the market but decreases after reaching a peak. The effect of relative market liquidity from hedgers on the difference in implied volatility spread is even more complex. Notice, however, that these characteristics are very similar to what is shown for the implied volatility smile and the difference in information asymmetry in Figures 10 and 11. Intuitively, as the relative information asymmetry converges between the two contracts, the degree of information risk that market makers face in the two contracts becomes very similar; because the risk that market maker has exposure to is essentially only the volatility risk, the bid-ask spread obtained using implied volatility as the measurement of option expensiveness is well suited to reflect the compensation for the market maker's exposure to such risk. However, due to the non-linearity of option prices across strike prices, the cheaper absolute price of the OTM contracts is much more sensitive to percentage measurement, meaning that a small change in price could lead to a large change in terms of percentage. While the difference in volatility information risk between contracts reduces as α increases, the absolute level of information actually increases in both contracts, as does the price of both contracts. Therefore, the higher sensitivity for the much cheaper OTM contract could have its spread affected relatively more than the ATM contract (percentage-wise), despite the fact that the difference in the risk has been reduced.

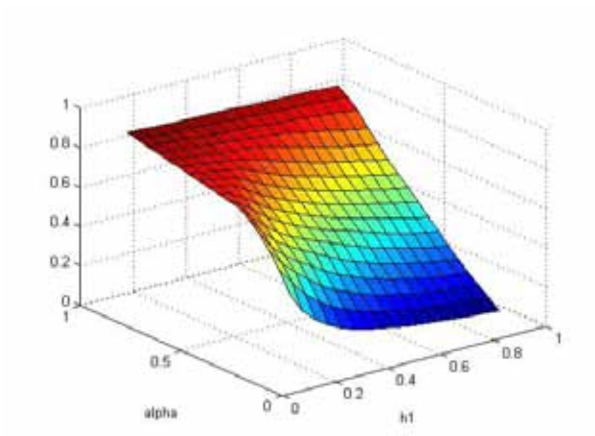

Figure 17. Difference in Percentage Spread between Contract-1 and Contract-2

This figure shows the predicted percentage spread of the two option contracts - $Spread_1\% - Spread_2\%$. The vertical z-axis is the size of spread difference. Left horizontal axis is the value of α; right horizontal axis is the value of h_1.

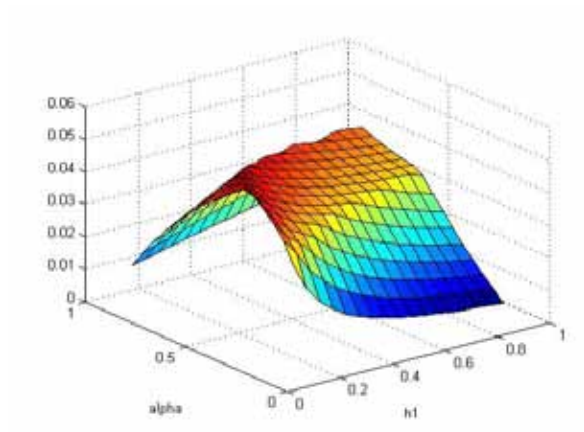

Figure 18. Difference in Implied Volatility Spread between Contract-1 and Contract-2

This figure shows the predicted implied volatility spread of the two option contracts - $(a_1 - b_1) - (a_2 - b_2)$. The vertical z-axis is the size of spread difference. Left horizontal axis is the value of α; right horizontal axis is the value of h_1.

CHAPTER 3 Behavior of Volatility Traders & Heterogeneous Information Asymmetry

3.1 *Introduction*

Black (1975) is the first to suggest the idea that an informed trader could take advantage of the higher leverage in option markets when making investment decisions. Since then, a number of related topics have attracted increasing attention in the literature. Firstly, there is this an active discussion over whether directional information investors trade in option markets and the extent to which this affects the pricing discovery and the market microstructure between the two markets [Stephan and Whaley (1990), Amin and Lee (1997), Easley, O' Hara, and Srinivas (1998), Chan, Chung, and Fong (2002), Chakravarty, Gulen, and Mayhew (2004), Cao, Chen, and Griffin (2005), and Pan and Poteshman (2006)]]. While the general consensus among these studies seems to agree on the existence of informed directional traders in option markets, the results are mixed in regard to their ability to influence option prices. Secondly, another area that has generated many thoughts is the research regarding the behavior of informed investors in the option markets. What makes it a fascinating topic is that in option market there are numerous option contracts for the informed traders to choose from, and each contract has its unique characteristics while still being linked to one another very closely. Easley, O' Hara, and Srinivas (1998) provide a theoretical framework suggesting that informed traders will optimize their investment by considering the tradeoffs between leverage, liquidity, and transaction cost. They demonstrate a number of conditions under which a 'pooling' equilibrium will exist, a case where informed directional investors trade in both stock and option markets. However, current empirical studies on this topic are at best inconclusive. For example, evidence from Kaul, Nimalendran, and Zhang (2004) and Chakraverty, Gulen and Mayhew (2005) leads to completely opposite conclusions. However, many of these works aim to better understand the market microstructure of option markets by investigating the role of directional information traders in the options market. Because the existence of volatility information trading [Ni, Pan, Poteshman (2008)] presents a unique type of volatility risk that cannot be easily hedged by the market makers,[14] such risk should play a more pivotal role in the pricing the efficiency of option markets than directional information risk. Therefore, it seems necessary to take a more systematic approach to examine the relationship between volatility information trading and the market microstructure of option markets to obtain a more nuanced overall picture.

As discussed in Chapter 2, my theory has very interesting implications for the behavior of informed volatility traders and consequently the structure of information asymmetry in option markets. A fundamental feature of options is that for two option contracts with the same underlying stock but varying strike prices, the Out of The Money (OTM) options create higher implicit leverage for informed volatility traders. There is a natural incentive for them to utilize as much of this higher leverage as possible if market conditions cooperate. So high liquidity in those these OTM contracts is crucial for the informed traders to take full advantage of the higher leverage. If the liquidity is insufficient for the informed traders to hide their orders, market makers will deem the order flow to be too toxic and thus increase the price to protect their own interests. The model predicts that in equilibrium, information asymmetry will always be higher for the OTM option contract, even though there are potentially two possible types of equilibrium. In the case of boundary equilibrium, the OTM option has enough liquidity to accommodate the volume of informed trading; there will be no information asymmetry for options with a strike that is closer to the stock price. If the market is in an interior equilibrium, informed volatility

[14] Jameson and Wilhelm (1992) conjectures that volatility risk is a unique risk for market makers in option markets because volatility uncertainty affects their ability to continuously rebalance their positions.

traders have to split their strategies between contracts to maximize their expected profit. However, the optimal strategy of the informed trader will also result in an equilibrium that information asymmetry in the OTM contract is higher. If this prediction is proved to be empirically true, it provides a direct theoretical foundation for and is consistent with evidence documented in a number of previous empirical studies. For example, Pan and Poteshman (2006) find that the same type of information embedded in more OTM option contracts has greater predictive power than in ATM options. This is only true if the degree of information asymmetry is greater for OTM options, as predicted by the model in this paper.

In this section, I attempt to contribute to the current literature by empirically testing the model's prediction on the behavior of volatility information traders in option markets as they try to strategically maximize their 'leverage' across contracts against market makers. Moreover, unlike most of the previous empirical studies (in which the behavior of informed trader is indirectly inferred from examining price discovery processes or spread movements), I utilize the theoretical model I established in the previous chapter (as well as intra-day trading data from the Chicago Board Option Exchange (CBOE)) to address this issue by directly estimating the level of information asymmetry embedded in the option markets. More importantly, the structure of the model allows me to separate the potential heterogeneity of information risk across contracts of different strike prices, which helps us to better understand the relationship between the strategic behavior of informed volatility traders, information asymmetry, and the numerous pricing features in the option markets. The estimation methodology is closely related to the work by Easley, Kiefer, and O'Hara (1996) and (1997), and Easley, Lopez de Prado, and O'Hara (2012); I discuss the details of this method in later sections.

I find that the empirical results regarding the structure of information risk within the option series is largely consistent with the theoretical prediction from the previous chapter. The OTM options, on average, have a higher degree of information asymmetry than the ATM options.

3.2 *The VPIN Variable for Option Market*

A direct test on the behavior of informed volatility traders and the equilibrium information risk structure of option markets would be estimating the degree of information asymmetry embedded in various groups of option contracts. The traditional approach for estimating information asymmetry in market microstructure research typically involves a maximum likelihood approach.[15] This method estimates the parameter values associated with the structural model to obtain the Probability of Informed Trading (PIN) variable. More recently, Easley, Lopez de Prado, and O'Hara (2012) have developed a new methodology to estimate information asymmetry (or flow toxicity) in a high frequency trading environment. They use a non-parametric approach to estimate the Volume-synchronized Probability of Informed Trading (VPIN).[16]. I am going to adapt this VPIN approach to estimate the probability of informed volatility trading in the option markets. Among its many advantages over the likelihood estimation approach, some of them are particularly relevant in light of this model. Firstly, the non-parametric feature of VPIN calculation is extremely appealing in this case because the number of parameters in the structural model is too many for the conventional likelihood estimation. Secondly, the VPIN methodology allows for a time-varying estimation of information asymmetry, whereas the conventional maximum likelihood approach assumes stable

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[15] Please see Easley, Kiefer, and O'Hara (1996) and (1997) for the details of this estimation method.
[16] Patent has been applied for their VPIN calculation algorithm.

parameter values over a certain period. This is particularly useful in later sections when I attempt to empirically examine the relationship between option prices and information asymmetry. Last but not least, the model in this paper does not specify the regularity of the information event, nor the time interval between the beginning of the game and when the volatility information is revealed to the public. Volatility information in the market can occur irregularly, and the extent to which it draws market attention can also vary. Such information can depend on the level of surprise of a data or earnings announcement, in which case informed traders start to gradually inject flows into the market a few days ahead of the announcement. Other volatility information can be related to market activities and sentiments; in this case, the realization of such information might be available to the public within the day. The fact that VPIN uses a volume-bucketing rule means that more weight is assigned to information associated with higher volume and that volatility information will be weighed in as it happens rather than become diluted over a specified time interval. For instance, consider two sets of options with two different underlying stocks; one of them has information embedded in trading volume on a daily basis, while the other one has either multiple information based trading episodes during a day or no information trading for many days. The manifestation of the former scenario would be stable daily volume across days; for the latter, one would see very large volume in days where information trading occurs but quiet trading activities for the other days. If I were to calculate the average probability of information-based trading over one month, the volume bucketing rule will provide a more consistent estimation between the two sets of options than using clock time as a basis.

3.3 *Estimation Methodology*

Recall that in Chapter 2, the probability of informed trading for each option category of contracts can be characterized by model parameters as

$\frac{\alpha\rho_i^*}{\alpha\rho_i^*+2(1-\alpha)h_i} = \frac{\mathrm{E[order\ imbalance\ for\ contract\ i]}}{\mathrm{E[total\ order\ size\ for\ contract\ i]}}$. Moreover, similar to most of information-based market microstructure models, the source of this expected order imbalance comes directly from the presence of the informed investor; in this model, the informed volatility trader. Follow the work of Easley, Lopez de Prado, and O'Hara (2012), the expected value of this imbalance for a given trading period should be $\mathrm{E}\left[\left|\mathrm{BUYS}_\tau^{\mathrm{i}} - \mathrm{SELLS}_\tau^{\mathrm{i}}\right|\right]$ as shown by Easley, Engle, O'Hara, and Wu (2008). Furthermore, because the expected total order size is simply the sum of Buy and Sell orders, we can express the VPIN for each option contract as:

$$VPIN = \frac{\mathrm{E}\left[\left|\mathrm{BUYS}_\tau^{\mathrm{i}} - \mathrm{SELLS}_\tau^{\mathrm{i}}\right|\right]}{\mathrm{E}\left[\mathrm{BUYS}_\tau^{\mathrm{i}} + \mathrm{SELLS}_\tau^{\mathrm{i}}\right]} \approx \frac{\sum_{\tau=1}^{\mathrm{n}}\left|\mathrm{BUYS}_\tau^{\mathrm{i}} - \mathrm{SELLS}_\tau^{\mathrm{i}}\right|}{\mathrm{nV}} \quad (28)$$

Because VPIN adopts a volume bucketing rule, τ is the index for each of the equally sized volume buckets, and the expected sum of Buy and Sell orders is fixed as V for each period.

Easley, Lopez de Prado, and O'Hara (2012) also introduced a new method to classify Buy and Sell volumes in the high-frequency trading environment. Although trading activity in the option markets has been expanding rapidly, it was still far from a high-frequency trading venue in 2003. Many of the difficulties in identifying Buy or Sell volumes in a high frequency world associated with using conventional approaches is therefore unlikely to persist in the option markets. Therefore, I will adopt a slightly modified Lee-Ready algorithm (1991) for the classification of non-market maker orders.

The size of each volume bucket V and the number of baskets n for each estimation period needs to be exogenously determined. Ideally, I would like to have sufficiently high daily trading volumes such that both n and V can be chosen to be reasonably large for representing a one-day equivalent estimate of the VPIN. For example, if n equals 50 and V is one-fiftieth of the average daily volume for a particular set of option contracts, then on a day of average volume the VPIN at the closing could effectively correspond to the daily probability of information-based trading. Unfortunately in this case, because the options market is not a high frequency environment, the average daily volume is relatively small, with large deviations across different equities. To illustrate the estimation results, I use $V = average\ daily\ volume, n = 40$ as an initial specification.[17]

In order to estimate the VPIN for the options market, option contracts are divided into two moneyness categories, according to their strike price and the prevailing underlying stock's price at any moment in time. For Call options, if $-0.05 < \frac{K}{S} - 1 < 0.1$, they are in the At-The-Money (ATM) category (or Near-The-Money (NTM) category); if $\frac{K}{S} - 1 \geq 0.1$, these options belong to the higher leveraged, Out-of-The-Money (OTM) category. Following the same principle, if $-0.05 < 1 - \frac{K_i}{S} < 0.1$ for Put options, these are in the ATM or NTM category; if $1 - \frac{K_i}{S} \geq 0.1$, these would be OTM Put options.

3.4 *Data & Sample Selection*

To estimate the VPIN of the options market, I will primarily use two intra-day datasets from the Chicago Board Option Exchange (CBOE): tick-by-tick quotes data and trades data, both including the 45 most active equity option series over the period from January to June 2003. The datasets cover the entire range of individual option contracts in various types, strike prices, and

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[17] There is no specific reason for this choice of parameters other than that I feel this combination will provide the best balance between the number and the quality of estimations. Easley, Lopez de Prado, and O'Hara (2012) use a combination of V equal to one-fiftieth of daily volume and n equal to 40.

maturities. The trades dataset also includes the underlying stock price at the time of each trade, but for two months this variable is missing. I replace the missing portion of this variable by merging NYSE TAQ data with the option data.

The sample will only contain contracts that have expiration dates of less than eight months because the trading activity significantly decreases as the duration of the contracts is extended. Another feature of option markets is that the trading activity explodes as a security draws near its maturity, ; both investors and market makers trade heavily to rebalance their portfolio and inventory positions before option settlement. Therefore, as a common practice, securities that expire in less than 5 days are excluded from the sample. Finally, since the trading activities appeared to be quite low in the data, despite they are being the most traded group of options during the period, if a particular group of options has fewer than 60 trading days with at least one transaction recorded during a day, the entire set of options with the same underlying equity will be excluded from the VPIN calculation.

Trading orders are identified as Buy or Sell orders using a reduced version of the Lee-Ready Algorithm against the quotes data. Specifically, if the trade price is above the mid-point of the prevailing quotes, it is treated as a buy order, and as a sell order if it is below the mid-point. When the trade price happens to be equal to the mid-point, the order is treated differently from the Lee-Ready Algorithm, which would compare the current trade price with the previous one.[18] Since the price of an option as a derivative can be affected by many factors such as underlying price and time between trades, it is entirely possible that the second stage of Lee-Ready algorithm might create more misclassifications if it is applied directly to the options market.

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[18] Specifically, if the tick price increases, it is assumed that the trade is Buy order initiated,; and if tick price decreases, the trade is considered to be a Sell order; if the price is unchanged from the previous one, then this process continues until a change of price is found.

Therefore, in this paper, whenever a trade price is equal to the mid-point it will be excluded from the sample. However, in market microstructure literature as well as for practitioners, precisely matching trades records with quotes data has always been more of an art than a science. Fortunately, options trading has yet to become a high-frequency world. The prevailing quotes are paired up with each transaction by the interval of minutes. Average bid and ask prices are used if multiple quotes were available during the same minute interval; however, a brief examination of the dataset suggests this is indeed a rare scenario.

In the final sample, the maximum number of trading days available for any option category is 125 trading days, using the parameter combination of $V = average\ daily\ volume, n = 40$, implies that a maximum of 125 volume-baskets would be created for an option category that recorded a transaction every single day during the sample period.[19] Therefore, this option category will have approximately 85 VPIN estimates. Table 2 provides a summary of the data used in the VPIN estimation.

[19] There are no specific reasons for the choice of parameter values other than trying to balance the total number of estimated VPINs and the quality of estimation. Reducing V and n increases the total number of estimates at the cost of increasing the noise in each estimation. The combination of average daily volume and 40 buckets is consistent with the parameter choice in Easley, Lopez de Prado, and O'Hara (2012). When estimation is performed using different parameter combinations, none of the qualitative conclusions from the presented results are affected.

Table 2. Summary of VPIN Estimation

This table reports values from January 2003 to June 2003 of a number of variables for different option types and categories. Only option categories with more than 60 trading days with at least one transaction recorded during a day are included. Options with maturities of less than 5 days and more than 8 months at the beginning of the day are removed from the sample. Number of VPINs to be estimated are based on parameter combination of $V = average\ daily\ volume, n = 40$.

	Puts		Calls	
	OTM	*ATM*	*ATM*	*OTM*
No. of stocks	*41*	*41*	*42*	*42*
Avg. No. of trading days	*115.2*	*119.6*	*119.3*	*122.1*
Avg. daily volume	*444.0*	*852.0*	*1562.9*	*664.4*
No. of transactions	*98341*	*258738*	*573543*	*227497*
Avg. moneyness (K/S)	*81.2%*	*97.5%*	*102.7%*	*123.9%*
Avg. maturity	*92.8*	*53.3*	*63.1*	*115.9*
No. of VPINs estimated	*3084*	*3262*	*3330*	*3448*

3.5 ***Results***

The VPIN calculation procedure mostly follows the algorithm defined in Easley, Lopez de Prado, and O'Hara (2012); minor adjustments are made to accommodate the structure of the data in this model. As stated earlier, I will present the results with V as the daily average volume for each option category and $n = 40$. This parameter choice is equivalent to saying that each VPIN is calculated using two months of trading data on average, but of course when information events arrive frequently during certain clock-time periods, VPINs are updated much more quickly. A summary of VPIN estimations using the various combinations of parameters $n\ and\ V$ can be found in the appendix; the results only differ quantitatively rather than qualitatively.[20]

[20] I also attempt to estimate the VPINs by using the combined average daily volume of each series as the bucket size for both moneyness categories; the estimation results are still qualitatively consistent with the conclusion.

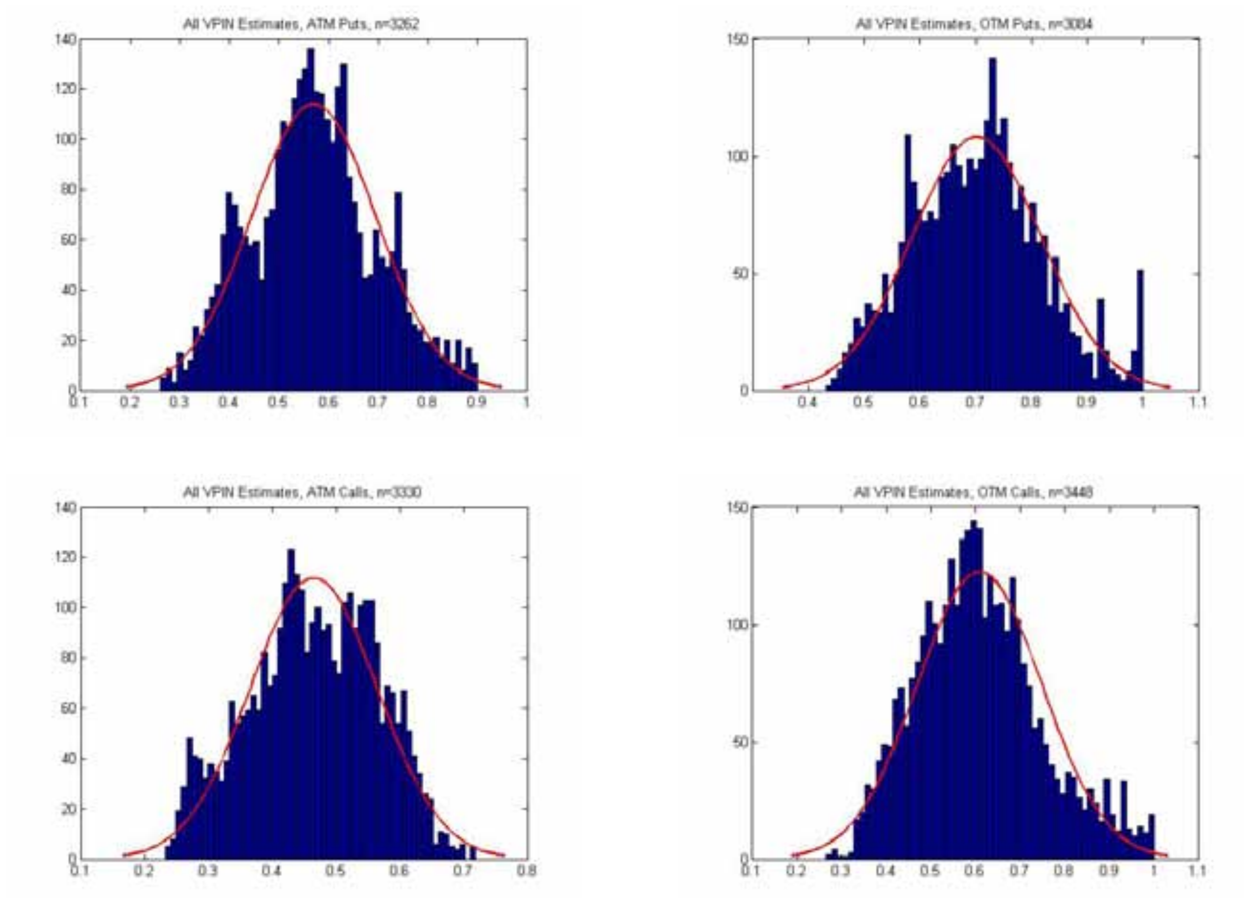

Figure 19. Distribution of VPIN Estimates

In this figure, the distribution of all VPIN estimates for each option category is presented in these four histograms. The parameters used for every option category is V= average daily volume, n=40.

As stated earlier, the VPIN can be viewed as a direct approximation of the probability of informed trading in each option category. Figure 19 shows the fitted distributions of VPIN estimates for each option category over the entire sample. The distribution of VPINs for the OTM category appears to have a higher mean for both option types; this is consistent with the hypothesis that, in equilibrium, OTM option contracts in general contain higher volatility information risk than ATM contracts. To determine whether this distribution feature over the entire sample is a result of a few individual option series, Figure 20 demonstrates a comparison between the volatility information risk embedded in OTM and ATM contracts of each stock. Each point corresponds to a particular stock, with the vertical axis measuring the average VPIN of its OTM contracts and horizontal axis being the average VPIN of the ATM contracts. A 45-degree line is plotted on both graphs; a point on the left hand side of the line indicates that the volatility information risk is higher in OTM options than ATM options for that particular stock. The fact that the majority of the points lie on the northwest side of the 45-degree line reinforces the conclusion that OTM options indeed have higher information risk than ATM options for most underlying stocks.

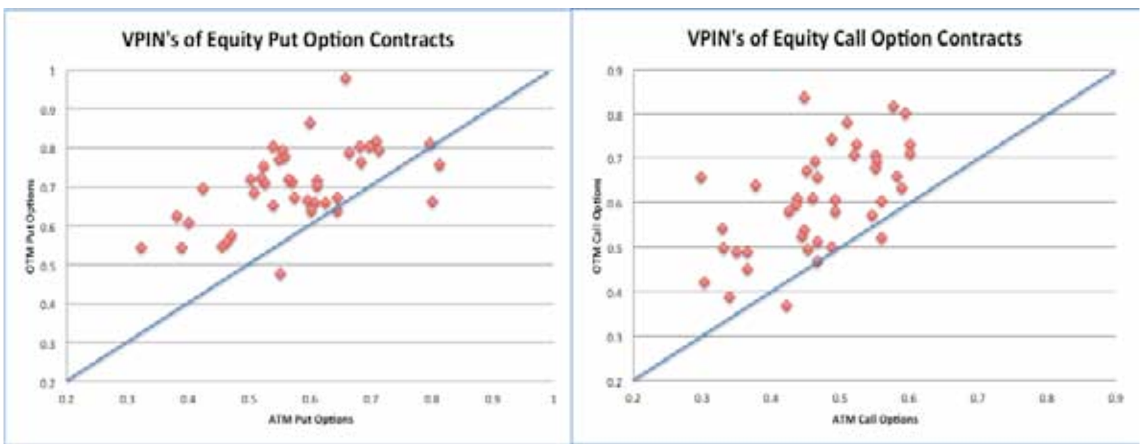

Figure 20. OTM VPIN vs. ATM VPIN

This figure shows the relationship between OTM VPINs and ATM VPINs of the same underlying stock. Each point represents one stock, its vertical coordinate represents the mean of OTM VPIN estimates, and its horizontal coordinate represents the mean of its ATM VPIN estimates.

To statistically examine this hypothesis, Panel A of Table 3 represents the distribution summary of average VPIN estimates for both OTM and ATM options across 45 stocks. Notice that regardless of the type of options, the OTM category on average has a much higher information risk as measured by the mean and median of VPIN estimates. I subsequently perform a non-parametric Mann-Whitney test on the hypothesis that the average VPIN estimates for OTM options have a higher median than VPIN estimates for ATM options; the results are shown in Panel B. The p-values strongly reject the null hypothesis in favor of the alternative, and it clearly indicates that this VPIN estimate is indeed consistent with the hypothesis that trading OTM options involves higher information risk than ATM options.

Table 3. VPIN Estimates and Mann-Whitney Tests

Panel A presents the means, medians, and standard deviations of VPIN estimates by option type (Put/Call) and category. Each VPIN estimate is calculated using V= average daily volume & n=40. Panel B shows the results from a Mann-Whitney test with a null hypothesis being both samples from the same distribution, which examines whether the distribution of VPIN for OTM options has a higher median than the distribution of VPIN for ATM options.

Panel A: VPIN Estimate Summary for all Option Categories

	Puts		*Calls*	
	OTM	ATM	ATM	OTM
Sample Size	41	41	42	42
Mean	0.7034	0.5765	0.4698	0.6074
Median	0.7103	0.5705	0.4665	0.6092
S.D.	0.0997	0.1135	0.0865	0.1168

Panel B: Mann-Whitney Test on Option VPIN's

Sample Size	m=41, n=41	m=42, n=42
P-Value	2.58E-06	2.50E-07

One potential concern of these VPIN estimates is that they might be subject to some form of volume bias. Since the sample consists of the most active equity options in terms of total trading volume, it is reasonable to claim that information risk as measured by VPIN in OTM contracts is higher than ATM contracts is a manifestation of the total trading volume, which is an element not modeled in the theoretical framework. To address this issue, Figure 21 plots the differences in VPINs between contracts against the average daily total trading volumes. Clearly, both the graph and the t-stat of 0.97 on the slope coefficient strongly suggest that the heterogeneous information risk captured by VPIN is robust to the size of the overall option trading activity.

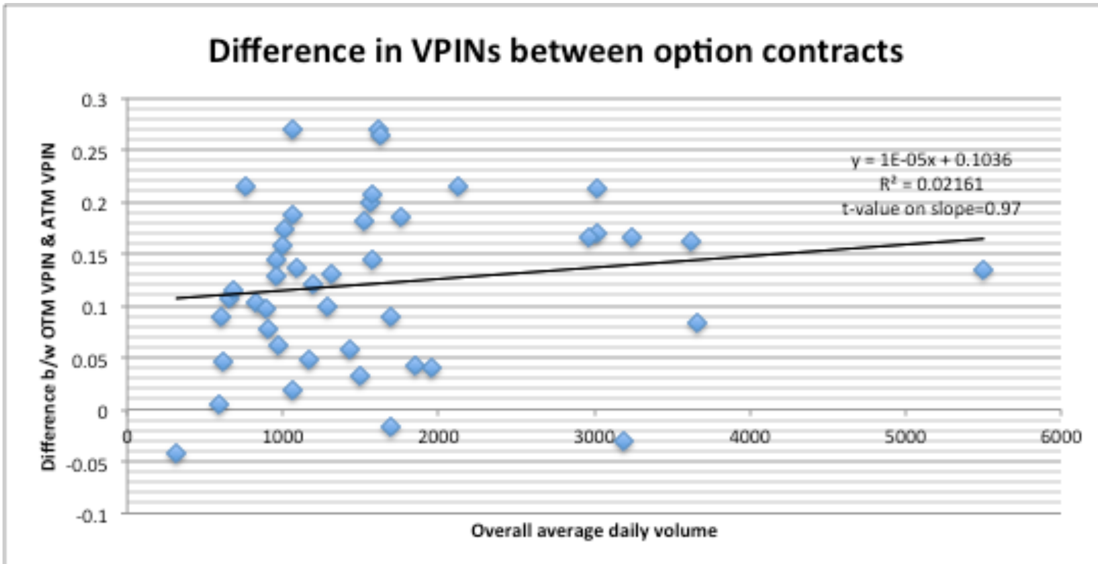

Figure 21. Difference in VPINs between Option Contracts

This figure plots the difference in average VPIN between the OTM and ATM contracts of each stock against the total daily volume of each option series. The equation represents the fitted line and the t-stat on the slope of this line is shown in the top-right corner.

Of course, we do have to keep in mind that these estimates may potentially over-estimate the level of volatility information risk if informed directional traders also bring significant volume. However, as stated earlier, both media articles and academic literature point to the idea that option markets are more likely to be dominated by volatility trading. More importantly, since market makers have the ability to hedge directional risk, the presence of informed directional traders should have very limited impact on the spread and pricing of options in a competitive environment. Therefore, their existence, and thus their trading volume, should only create noise in the process of estimating volatility information risk, which is a risk that is more difficult to be hedged by market makers. In the following sections where I analyze the impact of volatility information asymmetry on spread and implied volatility, I present several results that make convincing cases that these VPIN estimates are reasonably good proxies for reflecting the *relative* level of information asymmetry with respect to volatility.

CHAPTER 4 Volatility Trading & Option Market Microstructure

4.1 *Introduction*

It is widely recognized that inventory positions and asymmetric information are two major sources of risk associated with financial market dealers. The case of inventory position risk for equity dealers has been illustrated both theoretically and empirically in existing literature [for example, Ho and Stoll (1983) and Stoll (1978)b]. On the other hand, papers such as Glosten and Milgrom (1985) and Easley and O'Hara (1987) demonstrate why the existence of information asymmetry about future stock values between equity market makers and informed investors would result in market makers increasing bid-ask spreads to protect themselves from such risk. Despite the mixed empirical results on the behavior of informed directional investors in option markets, the complication is the extent to which these risks, with respect to the directional movement of the underlying asset, can be hedged away by an attempt to remain a delta-neutral position: a common practice with options market makers.

Some research have attempted to explain the market microstructure in option markets by using market frictions, with some success. For example, Hull and White (1987) and Jameson and Wilhelm (1992) use option derivatives such as Gamma and Vega to proxy for various factors associated with a market maker's inability to continuously rebalance their positions. Despite their serious efforts, market friction alone seems to capture only part of the spread behaviors. For example, to the extent that limits of arbitrage affect the bid-ask spread in option markets, it is not clear why OTM contracts, which require minimal positions to remain delta-neutral, should have a much higher spread size than ATM options.[21]

To the best of my knowledge, other than a few theoretical attempts to link option spreads

[21] For US equity option markets, the proportional spread for OTM contracts in general is greater than ATM options.

with volatility information risk, very little has been done to empirically explore the influence of volatility information asymmetry on option bid-ask spreads. Because the model developed in Chapter 2 utilizes an asymmetric information framework, the theory is able to generate specific predictions on the relationship between option spreads and volatility information risk. In this section, I empirically investigate the prediction on the role of volatility information trading in determining option spreads. I find that volatility information trading is a major source of risk for option market makers, as it accounts for a large proportion of variations in option spreads both cross-sectionally and dynamically. Moreover, I show that the potential heterogeneity in volatility information risk within the same option series also plays an important role in explaining the difference between OTM and ATM spreads.

Another contribution of this part of the dissertation is the ability to directly measure the level of information asymmetry using the theoretical framework and unique dataset. Many previous empirical works relied on the various measurements of implicit leverage as a proxy for information asymmetry, based on the notion that higher leverage attracts more informed investors. However, well-accepted theories such as those of Glosten and Milgrom (1985) and Easley and O'Hara (1987) suggest that the real genesis of information risk is not the absolute level of informed traders in a financial market; rather, it is the relative proportion between informed and uninformed traders that most concerns market makers. Using VPIN as a direct proxy for information asymmetry conveniently solves this problem, and it certainly helps us to better understand the relationship between volatility information trading and option market microstructure.

4.2 *Empirical Specification*

According to the theoretical results from Chapter 2, the spread of an option contract is derived as the following:

$$\begin{aligned} Spread_i = & \left(\frac{\alpha\rho_i^{B^*} + (1-\alpha)h_i}{\alpha\rho_i^{B^*} + 2(1-\alpha)h_i} * C_i(\overline{\sigma}) + \frac{(1-\alpha)h_i}{\alpha\rho_i^{B^*} + 2(1-\alpha)h_i} * C_i(\underline{\sigma}) \right) \\ & - \left(\frac{(1-\alpha)h_i}{\alpha\rho_i^{S^*} + 2(1-\alpha)h_i} * C_i(\overline{\sigma}) + \frac{\alpha\rho_i^{S^*} + (1-\alpha)h_i}{\alpha\rho_i^{S^*} + 2(1-\alpha)h_i} * C_i(\underline{\sigma}) \right) \end{aligned} \qquad (29)$$

If we assume that in equilibrium, the difference in volatility risk faced by market makers between executing a buy order and executing a sell order is sufficiently small, the spread can be expressed as:

$$Spread_i \approx \left[C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right] \cdot PIN_i^{vol} \qquad (30)$$

$$Spread\%_i \approx \left[\frac{C_i(\overline{\sigma}) - C_i(\underline{\sigma})}{\frac{1}{2}\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)}\right] \cdot PIN_i^{vol} \qquad (31)$$

What this relationship essentially suggests is that the spread of any option contract at a given time is determined by the product of the relative difference in option values at each volatility state and the volatility information risk of the corresponding contract. If a conditional lognormal distribution is the equilibrium common belief in the market once true volatility is revealed, that is, the option's value is determined by the Black-Scholes formula in the second period, then by assuming the volatility parameters in the model as simple linear functions of the underlying's historical realized volatility, the percentage spread of an option contract may be expressed as:

$$Spread\%_i = \left[\frac{BS_i(K_i, \sigma_H) - BS_i(K_i, \sigma_L)}{\frac{1}{2}\left(BS_i(K_i, \sigma_H) + BS_i(K_i, \sigma_L)\right)}\right] \cdot PIN_i \tag{32}$$

where $\sigma_H = \sigma^{realized} * 1.2$ $\sigma_L = \sigma^{realized} * 0.8$, and $BS(K_i, \sigma_s)$ is the Black Scholes option value if it has a strike of K_i and the future volatility is σ_s.

In order to empirically examine the role of information risk in an option's spread as characterized above, I analyze the daily market-opening proportional spread[22] of option contracts. The advantage of using the market-opening spread is that it is a point of time when minimal public information and trading patterns are available to the market maker during a normal trading day. Therefore, it provides a great snapshot of the option markets that is closest to the scenario described in the static theoretical model, where beliefs are formed based on exogenous factors and historical data. Additionally, because a daily estimate of PIN is required for this analysis, there are three issues that need to be addressed. Firstly, one should notice that when a VPIN is obtained as a direct estimate for the Probability of Information Trading in each option group, it is calculated through trading activities on a volume-clock basis, which means some trading days in the sample may not have a VPIN estimate; and other days may have multiple VPIN estimates. Therefore, I am going to construct a time series of previously estimated VPINs to correspond with the average daily spread. Simply put, the day in which the first VPIN is estimated for the option category will be the first daily VPIN observation. If more than one VPIN is estimated during a day, the last estimate will be taken; if there does not exists a single

[22] Due to the non-linearity in Black-Sholes price's sensitivity to volatility, and the fact that volatility parameters in the model are not directly observable, any systematic incorrect assumption on the volatility parameters value could lead to large non-linear variations in the predicted dollar spread for options. To this end, since the volatility parameters are assumed to take the same simplistic linear form across options, using a proportional spread will help to reduce this misspecification risk. When the same test is performed using a dollar spread, the statistical and economic significance of independent variables remain very similar to the presented results, but the R-square becomes much smaller.

VPIN estimate for a trading day, I use the VPIN from the previous day as the best estimate of the day. Secondly, because the daily VPIN is constructed based on the information at the end of each trading day, the best estimate of information risk a market maker has at the market opening is the VPIN estimate from $t-1$. Finally, due to the limitation of data, VPINs are only estimated for two moneyness categories of each option series. In order to carry out this analysis, I take the average spread across contracts within the same moneyness group, and then investigate the average proportional spread of each option moneyness group. Therefore, the main equation I will utilize to examine the model's implication on the option's spread is the following:

$$Spread\%_{m,t} = \beta_0 + \beta_1 \left[\frac{1}{N_{m,t}} \sum\nolimits_{i \in m} \frac{BS_t(K_i, \sigma_H) - BS_t(K_i, \sigma_L)}{\frac{1}{2}\left(BS_i(K_i, \sigma_H) + BS_i(K_i, \sigma_L)\right)} \right] \cdot VPIN_{m,t-1} + \epsilon_{m,t} \qquad (33)$$

where m denotes the moneyness group, $\text{i.e.}, m \in \{OTM, ATM\}$; $N_{m,t}$ is the number of contracts available in moneyness group m at time t.

Based on the explanation in the previous section, an intuitive way to interpret this relationship is that the average opening proportional-spread for each option category is a linear function of the multiplicative product between the average percentage difference (in option values between two volatility states) and the best estimate of volatility information risk in the corresponding option group at the time.

In addition to volatility information risk, which is the only source of risk included in the theoretical model, there are other potential factors that could influence the market microstructure and the spread of the options market. To the best of my knowledge, the following are the most commonly discussed factors in the existing literature:

(a) Initial Hedging Cost (IH)

Option markets dealers face certain transaction costs such as the underlying stock's bid-ask spread whenever they take on and/or liquidate their delta-neutral hedging positions. This cost can be approximated by:

$$IH = \psi S \Delta \tag{34}$$

where ψ is the proportional spread of the underlying stock, S is the underlying stock price, and Δ is the option delta in the B-S-M formula.

(b) Discrete Rebalancing Cost (RC)

If the underlying stock's return follows a log-normal distribution with known volatility, an option market maker could in theory dynamically rebalance her portfolio to remain a delta-neutral position throughout the life of the option contract. However, due to the discrepancy between a theoretical frictionless market and the imperfect real world, it is impossible for option market makers to continuously rebalance positions without incurring extra cost. I define the discrete rebalancing cost in a manner similar to Leland (1985) and Boyle and Vorst (1992):

$$RC = 2\psi v \tag{35}$$

where ψ is the proportional spread of the underlying stock and v is the option Vega derived from the B-S-M formula. To simplify the calculation, a constant rebalancing period across options is assumed.

(c) Daily Transaction Volume (DV)

The expected trading volume of options should help to explain an option's spread in two ways. First, it serves as a good proxy for the average order processing cost for market makers. As order-processing costs tend to stay constant given a particular transaction, a larger expected trading volume should lead to a lower average order processing cost. Second, higher trading activity could also lead to a greater likelihood that option transactions will offset each other, thus reducing the need for market makers to rebalance their positions in the future. Under both scenarios, one should expect a negative relationship between the average daily volume of options and their spread level.

To control for these factors, each of the variables is added to the regression analysis. The average value of these variables is taken if the option contracts belong to the same moneyness category. Therefore, I examine the impact of volatility information risk on option spreads after controlling for other potential determinants of option spreads by analyzing the following equation:

$$
\begin{aligned}
Spread\%_{m,t} = \beta_0 \\
+ \beta_1 \left[\frac{1}{N_{m,t}} \sum_{i \in m} \frac{BS_t(K_i, \sigma_H) - BS_t(K_i, \sigma_L)}{BS_t(K_i, \sigma_H) + BS_t(K_i, \sigma_L)} \right] \\
\cdot VPIN_{m,t-1} + \beta_2 IH_{m,t} + \beta_3 RC_{m,t} + \beta_4 DV_m \\
+ \epsilon_{m,t}
\end{aligned}
\qquad (36)
$$

4.3 *Data & Sample Selection*

<u>*Spread Characteristics*</u>

Figure 22 shows the spread structure of two particular stock option series on a particular day before filtering the data for regression. What we see from the graph is that the average percentage spread is the highest in the most OTM Put options (more than 70% OTM); Call options also appear to have a larger spread as the strike price increases. Whereas in general, ATM options have the smallest spread for these two stocks. Another interesting characteristic of the figure is that the percentage spread structure also displays a ‘smile’ feature that one typically finds in the option’s implied volatility for these two stocks.

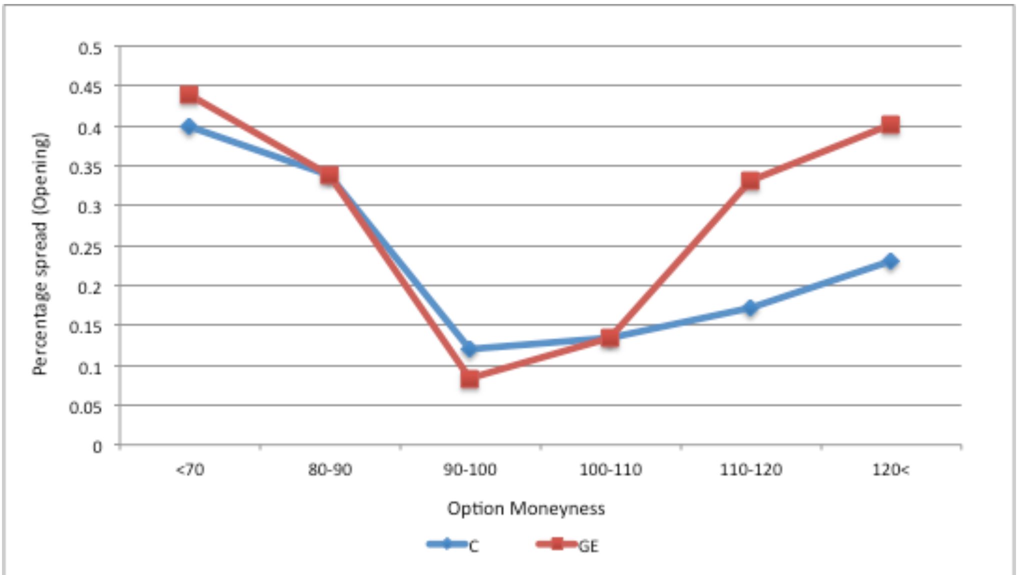

Figure 22. Spread Structure of Citi Options & GE Options on January 24, 2003.

This figure shows the average percentage spread of options in each moneyness category for two particular stock option series before filtering the data.

<u>Sample Selection</u>

To stay consistent with VPIN estimation, option contracts are divided into two categories: the At The Money (ATM) group consists of contracts that are between 5% In The Money and 10% Out of The Money (both inclusive); and anything greater than 10% is classified as the Out of The Money (OTM) group. Since option contracts with different underlying stocks may have different expiration dates throughout the calendar month, and the sensitivity of option values to time to maturity increases as the contract approaches its expiring date, I am going to only include contracts that have a 'next-month'[23] expiration in order to maintain a reasonable consistency across underlying assets and strike prices. To construct the volatility information risk factor for each available quoted contract ($\Sigma(\sigma)VPIN_t = \left[\frac{1}{N_{m,t}}\sum_{i\in m}\frac{BS_t(K_i,\sigma_H)-BS_t(K_i,\sigma_L)}{BS_t(K_i,\sigma_H)+BS_t(K_i,\sigma_L)}\right]\cdot VPIN_{m,t-1}$), a

[23] Specifically, each option series with the same underlying stock in the final sample will contain one expiration date—the second nearest expiration date available for each stock.

valid VPIN estimate must be available. Because the first estimated VPIN for each option category may not be available at the beginning of the sample period (unless 40 volume-baskets were accumulated on the first day), only those opening quotes with a valid VPIN estimate at the beginning of the day are included. Lastly, if the information related to the underlying stock, such as its price at $t-1$ or its historical volatility, is not available in the data for particular days, those days are excluded from the final sample. In order to calculate the volatility information risk and the control variables as discussed above, a standard no-dividend paying B-S-M formula and its derivatives, Delta and Vega, are used. Specifically, to calculate the opening Black-Scholes value of each option contract, as well as to determine the option category classification (i.e., whether an option is OTM or ATM at the opening), the stock's closing price from the previous trading day is considered to be the value of the underlying asset when the market opens. Among other parameters involved in the B-S-M formula, their prevailing values at the beginning of the day are applied to the price calculation. A simple equally weighted average is used when calculating the mean proportional spread for each moneyness category m, the volatility risk factor for that category, as well as all of the control variables. The daily volume variable DV_m is the average trading volume for category m of a particular stock obtained from the entire 6-month sample period. Because the primary focus of this dissertation is to study the behavior of a volatility information trader across strikes and its impact, theoretical and empirical differentials between Call and Put options are not explicitly addressed in the structural model. Therefore, the two option types are presented and tested separately.

4.4 *Summary Statistics*

There are a total of 9,518 'option category-day' of spread observations available after filtering the original data, including 4,651 Put observations. Table 4 provides the summary statistics of all the key variables involved in previous specifications. The actual market opening proportional spread (Spread%) has an aggregate mean of 0.38, or 38% of the option value (by mid-point). Among all the option contract groups observed in the sample, the maximum average spread size is 1, indicating that the size of the spread is never greater than the mid-point value of contracts in the observed data. The approximation of spread attributes to volatility information risk ($\Sigma(\sigma)VPIN$) has a distribution that is fairly close to the actual observed spread in terms of descriptive statistics, with the mean, minimum, and maximum slightly above the observed; the skewness and kurtosis are slightly smaller than the actual spread distribution. The minimum of both the Initial Hedging Cost (IH) and the Discrete Rebalancing Cost (RC) is zero, which suggests that the differences between the bid and ask prices of those active underlying stocks are very small relative to the absolute level of their price, thus translating to almost zero hedging frictions as defined earlier. On average, the transaction volume is lower for Put options than for Call options.

Table 4. Summary Statistics

This table reports summary statistics of the filtered sample. Spread% is calculated at market opening; proportion spread of the underlying asset is obtained from market closing data from previous day. Volatility information risk's contribution to spread, $\Sigma(\sigma)VPIN$ is calculated based on B-S formula and σ_H=1.2 times historical volatility and σ_L = 0.8 times historical volatility. Daily Volume is the average number of total contracts traded during the entire sample. Panels A through C show the summary statistics for Put options, Call options, and all types combined, respectively.

	Mean	Std.	Skew	Kurt	Min	Max
Panel A: Put Options						
Spread%	0.36	0.26	0.74	2.46	0.03	1.00
$\Sigma(\sigma)VPIN$	0.54	0.32	0.58	2.04	0.13	1.49
IH	0.02	0.03	3.64	22.48	0.00	0.31
RC	0.01	0.01	3.40	21.03	0.00	0.11
DV	724.01	621.67	1.88	6.34	47.59	2951.77
Panel B: Call Options						
Spread%	0.39	0.28	0.67	2.32	0.04	1.00
$\Sigma(\sigma)VPIN$	0.40	0.17	0.72	2.59	0.15	1.07
IH	0.02	0.03	3.19	18.78	0.00	0.36
RC	0.01	0.01	2.99	18.08	0.00	0.11
DV	954.04	840.38	1.78	6.43	12.00	4422.76
Panel C: Aggregate						
Spread%	0.38	0.27	0.71	2.40	0.03	1.00
$\Sigma(\sigma)VPIN$	0.47	0.26	1.03	3.26	0.13	1.49
IH	0.02	0.03	3.39	20.35	0.00	0.36
RC	0.01	0.01	3.18	19.43	0.00	0.11
DV	841.63	750.43	1.93	7.20	12.00	4422.76

Table 5 shows the filtered sample means of those variables in Table 4, as well as the B-S pricing features broken down by option types and moneyness categories.

Clearly, the spread size for OTM options is significantly larger than ATM options; this characteristic is also reflected in the construction of spread attributes to volatility information risk ($\Sigma(\sigma)VPIN$). $\Sigma(\sigma)VPIN$ is larger for OTM options for two reasons. Firstly, since the elasticity of change in option value with respect to change in volatility is higher when the strike price is lower (higher) for Put (Call) options; the relative price range $\Sigma(\sigma)$ is larger for OTM options. Secondly, because OTM contracts contain greater volatility information asymmetry, their VPIN is also higher; thus, the joint result for $\Sigma(\sigma)VPIN$. Additionally, as expected, ATM options have higher B-S Delta and Vega compared with OTM options. Subsequently, both the Initial Hedging Cost and the Rebalancing Cost is slightly higher for ATM options. The average trading volume appears to be greater for the ATM category in both option types. What is interesting is that both the per dollar Delta and Vega are much larger for OTM options, indicating that if the conditional B-S formula is used in the equilibrium, the OTM contracts on average provide a higher rate of return, or 'implicit leverage', for the informed volatility trader.

Table 5. Summary Statistics Broken Down by Option Type and Moneyness Category

This table reports summary statistics from January 2003 to June 2003 of a number of variables for different option types and categories. Spread% is calculated at market opening; proportion spread of the underlying asset is obtained from market closing data from previous day. Volatility information risk's contribution to spread, $\Sigma(\sigma)VPIN$ is calculated based on B-S formula and σ_H=1.2 times historical volatility and σ_L = 0.8 times historical volatility. Daily Volume is the average number of total contracts traded during the entire sample.

	Put		*Call*	
	OTM	ATM	ATM	OTM
Spread%	0.59	0.16	0.18	0.62
$\Sigma(\sigma)$VPIN	0.85	0.28	0.27	0.55
IH	0.01	0.02	0.03	0.01
RC	0.00	0.01	0.01	0.00
DV	483.75	931.62	1409.59	461.27
Delta	0.16	0.48	0.42	0.21
Vega	3.24	5.62	5.06	2.95
Delta/$	0.33	0.23	0.31	0.44
Vega/$	6.49	2.13	3.42	6.19

4.5 *Results*

There is clear evidence that volatility information risk plays a vital role in determining option spreads. As shown in column 1 of Table 6, the R-squared in the regression result (when including the volatility information risk) alone explains 62% of the variations in the opening proportional spread of option contracts. The estimated coefficient on the volatility information risk factor ($\Sigma(\sigma)VPIN$) has a t-statistic of 87.75; its value of 0.645 implies that whenever the model predicts a one percent increase in option spread due to a change in information asymmetry about volatility, the actual observed proportional spread will increase by 0.645 percentage points. Another interesting economic interpretation of this coefficient is that it suggests the contribution of heterogeneity in information risk between OTM and ATM contracts to their difference in proportional spread is approximately 0.082. To obtain this number, recall from the last section that, on average the VPIN of OTM Put contracts is greater than ATM Put contracts by 0.1269 (0.7034-0.5765=0.1269). Because the coefficient on $\Sigma(\sigma)VPIN$ is 0.645, the proportional spread difference between option moneyness categories of the same underlying stock is on average, **0.645***0.1269=0.082, larger than what would have been predicted if the level of volatility information risk is identical across the option strike prices. Columns 2-4 display the regression results when controlling for some or all of the other factors that may affect the option spreads as discussed earlier. The volatility information asymmetry term remains highly significant in all regressions even though both the coefficient and t-statistics reduce slightly as more independent variables are included, yet the minimum t-statistic is still 66.6. In fact, to interpret the economic significance of the coefficient on volatility information risk in the last column of Table 6, where all control variables are included, let us consider the average option group in the sample (i.e., when all the variables equal to their sample means). The volatility information risk ($\Sigma(\sigma)VPIN$) accounts for approximately 87.7% of its observed proportional opening spread.[24] Among the

[24] As shown in the summary statistics, the average Put option group has Spread%=0.36, Σ(σ)VPIN=0.54, IH=0.02, RC=0.01, DV=724.01. The estimated equation that fits this average group is Spread%=0.0781+0.584*Σ(σ)VPIN+(-2.471)*IH+0.5745*RC+(-4.35e-05)*DV; thus, the contribution of volatility information risk on the total spread is $\frac{0.584*\Sigma(\sigma)\text{VPIN}}{\text{Spread\%}}$, which equals 87.7%.

estimation results for control variables, the next largest contributor to the spread of the average option category is daily volume (DV). Moreover, its estimated coefficient is negative. It is consistent with the notion that more frequent trading activity will lower the average processing cost and reduce the need to rebalance positions—both would lead to smaller spread.

Table 6. Volatility Information Risk and Opening Spread of Put Options

	(1) Spread%	(2) Spread%	(3) Spread%	(4) Spread%
Σ(σ)VPIN	0.645**	0.634**	0.605**	0.584**
	(87.75)	*(82.15)*	*(74.76)*	*(66.60)*
IH		-0.444**		-2.471**
		(-4.993)		*(-5.951)*
RC			-1.225**	5.745**
			(-4.808)	*(4.794)*
DV			-4.34e-05**	-4.35e-05**
			(-10.78)	*(-10.83)*
Const	0.00849	0.0219**	0.0692**	0.0781**
	(1.833)	*(4.102)*	*(9.854)*	*(10.91)*
Observations	4,651	4,651	4,651	4,651
R-squared	0.624	0.626	0.634	0.637

This table reports the regression results of daily Put options spread from January 2003 to June 2003. The dependent variable is average proportional spread of each option category quoted on the CBOE at the market opening. Σ(σ)VPIN is the volatility information risk factor constructed, based on daily VPIN estimation and a linearly approximated range for underlying's future volatility (Σ(σ)). IH and RC are estimates of the Initial Hedging Cost and Discrete Rebalancing, based on underlying stock's spread and B-S option pricing characteristics. DV is the average daily volume of each option category during the sample period. T-statistics are shown in parentheses; the significance level of coefficients is shown according to criteria: ** $p<0.01$, * $p<0.05$.

Table 7 shows the regression results by estimating the spread specification for Call options. The key elements are essentially the same as the results from Table 6. The estimated coefficient of volatility information risk factor is statistically significant, and economically it also appears to be the clear dominant factor in determining the spread structure of Call options, both within and across the underlying stock. The regression R-squared is 0.546 in the full specification in column 4; although a large number, it is fractionally smaller than the result from the Put options.

However, it is interesting to note that for both Put and Call options, we see an alternating sign for rebalancing cost coefficient (RC), from -1.225 in column 3,when IH is excluded, to 5.5745 in column 4 in Table 6; and a negative coefficient of initial hedging cost (IH), which is the opposite of what one would expect. One possibility is that while initial hedging friction may be an important factor in explaining the cross-sectional difference in option spreads, but it suffers when intra-option series features are also included in the investigation. Within the same option series, because the OTM contracts have a smaller Delta and Vega, the cost related to the hedging action is therefore smaller for OTM options; this negative correlation with the observed intra-spread difference seems to be a contributing reason for these results, despite the empirical specification controlling for volatility information risk. Nonetheless, although the primary focus of this dissertation is on the impact of volatility information asymmetry, I believe that further study on the hedging frictions' impact on spreads should lead to more interesting conclusions.

Table 7. Volatility Information Risk and Opening Spread of Call Options

	(1) Spread%	(2) Spread%	(3) Spread%	(4) Spread%
Σ(σ)VPIN	1.187**	1.180**	1.051**	0.976**
	(72.18)	(69.52)	(56.66)	(48.27)
IH		-0.159		-3.535**
		(-1.685)		(-8.924)
RC			-0.127	10.56**
			(-0.445)	(8.580)
DV			-5.56e-05**	-5.36e-05**
			(-15.16)	(-14.72)
Const	-0.0838**	-0.0779**	0.0245*	0.0516**
	(-11.72)	(-9.777)	(2.353)	(4.789)
Observations	4,867	4,867	4,867	4,867
R-squared	0.517	0.517	0.539	0.546

This table reports the regression results of daily Call options spread from January 2003 to June 2003. The dependent variable is the average proportional spread of each option category quoted on the CBOE at the market opening. Σ(σ)VPIN is the volatility information risk factor constructed based on daily VPIN estimation and a linearly approximated range for underlying's future volatility (Σ(σ)). IH and RC are estimates of the Initial Hedging Cost and Discrete Rebalancing based on the underlying stock's spread and the B-S option pricing characteristics. DV is the average daily volume of each option category during the sample period. T-statistics are shown in parentheses; the significance level of coefficients is shown according to criteria: ** p<0.01, * p<0.05.

CHAPTER 5 Skewness of Volatility Smile and Information Asymmetry

5.1 *Introduction*

My theory predicts that when volatility information traders behave strategically as described in Chapter 2, option pricing features such as the implied volatility smile should be affected by the structure of information risk within the same option series. If various aspects of the market lead to a higher level of heterogeneity in information asymmetry between OTM and ATM contracts of the same underlying asset, not only will this have a direct effect on the IV skew as the relative expensiveness of OTM increases, it will also influence how change in volatility uncertainty affects the IV skew in equilibrium. To examine whether these predictions by the theoretical model are consistent with actual market outcomes, I am going to investigate three main hypotheses on the dynamics[25] of implied volatility skew in option markets:

a) The skewness of an option series of a particular asset should be positively related to the difference in information asymmetry between option contracts.

b) If volatility uncertainty is a linear function of historical volatility, a change in the historical realized volatility will directly affect the shape of the volatility skew.

c) The sensitivity of volatility uncertainty's impact on skewness depends on the heterogeneity in volatility information risk between OTM and ATM contracts.

Previous research on the empirical properties of Implied Volatility Function (IVF), or Implied Volatility Smile (IV smile), are mostly dominated by studies on the level or the slope of implied volatility in relation to the distribution of the underlying asset. For example, to demonstrate the necessary relationship between the observed IVF and the properties of the

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[25] Unfortunately, due to the limitation of data there is not a large enough sample to perform any meaningful cross-section analysis. It is undoubtedly very interesting for future research to also investigate the empirical implications of the model on the cross-section features of option skewness.

underlying asset, Anderson, Benzoni, and Lund (2002) show that the combination of randomly arriving jumps in security price and stochastic volatility are required to capture the time-series dynamics of index returns; Bates (2000) shows that including a jump process into a stochastic volatility model can help capture the dynamics of IVF to the extent that parameters must be set at unreasonable values, just to mention a couple[26]. In spite of the increasing effort, they fail to fully capture the slope and the dynamics of IVF. Other studies also attempt to explain the dynamics of IVF using a variety of existing factors in financial markets. Bollen and Whaley (2004), Gârleanu, Pedersen, and Poteshman (2009), and Nordén and Xu (2010) attempt to further study IVF by considering its relationship to the demand/liquidity in option markets. Peña, Rubio and Serna (1999) and Deuskar, Gupta, and Subrahmanyam (2008) investigate the economic determinants of the slope of IVF by using market microstructure variables. However, with the various levels of success in previous studies, to the best of my knowledge, there has been no research that directly examines the economic significance of a volatility trader's behavior and the structure of information asymmetry on the empirical dynamics of implied volatility smile.

In this chapter of the dissertation, in addition to testing whether the predictions my theoretical model are consistent with real world data, I also attempt to contribute to the empirical literature of options IV smile in the following way. First, using US equity options data from the Chicago Board Option Exchange (CBOE), I am the first to document that the difference (or heterogeneity) in volatility information risk across option strikes may have a significant impact on the shape of the volatility smile. Second, I extend the current understanding on the dynamics of the volatility smile by exploring the role of volatility trading in determining the shape of the

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[26] Bollen and Whaley (2004) provide a thorough review on the development of using option-pricing models to explain implied volatility smile.

IVF. Specifically, I present evidence that the interaction between the cross-section differences in the structure of volatility information risk and historical volatility can help to explain the time-series features of implied volatility smile in the US equity option market. Third, the consistency between the empirical results and the theoretical model demonstrates the value of heterogeneous volatility information risk as an important and alternative insight to option pricing that is worth further exploration. In particular, it may help to facilitate the understanding of puzzles such as the under-prediction of volatility skew by stochastic volatility pricing models.

5.2 *Empirical Specification*

First and foremost, I will define and explain the construction of the volatility skew variable. One way of measuring the shape of volatility skew is to impose structural implied volatility functions to obtain an estimate of the curvature.[27] But in Nordén and Xu (2010), they discuss that such approximation may suffer from several specification biases when the number of traded option contracts across moneyness is limited. In this paper, I am going to measure implied volatility skew following the spirit of its definition in the theoretical model, as the relative distance between OTM implied volatility and ATM implied volatility. Unlike the theoretical environment in which there are only two contracts available for each option series, the number of quoted option contracts that fall into the Out of The Money (OTM) or At The Money (ATM) category is much greater and varies both cross-sectionally and from time to time. In order to construct a robust and consistent IV skew measurement, I first measure the relative distance in terms of B-S implied volatility between two Put option contracts with the smallest and the largest

[27] Peña, Rubio and Serna (1999) and Deuskar, Gupta, and Subrahmanyam (2008) both adopt a method of this nature to estimate the curvature.

strike price among all available contracts belong to the predetermined OTM and ATM category.[28] To make each measurement of the skewness comparable to one another, I normalize the percentage difference by dividing the 'moneyness distance' between the two extreme contracts. Specifically, for each underlying asset on day t, the implied volatility skew is measured by:

$$Skew_t = \frac{\dfrac{IV_{OTM,t} - IV_{ATM,t}}{IV_{ATM,t}}}{\left|\dfrac{K_{OTM}}{S_t} - \dfrac{K_{ATM}}{S_t}\right|} \tag{37}$$

where K_{OTM} is the strike price of the *most* OTM contract quoted at day t; thus, $\frac{K_{OTM}}{S_t}$ is a measure of option moneyness for that contract at day t, and $IV_{OTM,t}$ is the B-S implied volatility of the OTM contract at day t. Analogously, K_{ATM}, $\frac{K_{ATM}}{S_t}$, and $IV_{ATM,t}$ are the values for the strike price, option moneyness, and B-S implied volatility of the ATM contract. It should be noted that due to the movements of the underlying's asset price, not only the level of individual implied volatilities is affected in a non-linear fashion, the strike price used in the skew calculation may also vary over time. However, the normalization approach ensures that the measurement skewness remains consistent and robust, both dynamically and across stocks.

To investigate the hypotheses on implied volatility skew generated by the theoretical predictions, I use the VPIN as the proxy for information asymmetry in the option market. Following the protocol of previous sections, I transfer the volume-bucket estimated VPIN into a daily VPIN measurement by taking the last estimate on days in which there are multiple estimates. For days without a valid estimate of VPIN, the value from previous trading day is used. However, in addition to the strong autocorrelation between the daily VPIN estimates due to

[28] The classification of the moneyness category is detailed in a later section.

the estimation methodology, it is also unclear how often the volatility information structure is likely to change and what is the frequency that market makers update their beliefs. Instead of examining the changes of skew on a daily basis, I will test the empirical implications of volatility information trading's impact on the volatility smile by estimating the following equation:

$$\Delta Skew_\tau = \beta_0 + \beta_1 \Delta\sigma_\tau^{hist} + \beta_2 \Delta VPINdiff_\tau + \beta_3 VPINdiff_\tau \cdot \Delta\sigma_\tau^{hist} + SR_\tau + ND_\tau + RND_\tau + RL_\tau + \epsilon_{i,\tau} \tag{38}$$

In equation (38), the subscript τ denotes an interval of a ten trading day period. $\Delta\sigma_\tau^{hist}$ denotes the change of historical volatility from period $\tau-1$ to τ. $\Delta VPINdiff_\tau = [VPIN_{OTM,\tau} - VPIN_{ATM,\tau}] - [VPIN_{OTM,\tau-1} - VPIN_{ATM,\tau-1}]$ is the change in the *difference* between the OTM options VPIN and the ATM options VPIN over each period. This variable essentially captures the importance of the volatility information trader's behavior in the option market by measuring the difference in volatility information risk between the higher rate of return OTM contracts and the ATM contracts. Notice that simply an increase in the participation by informed volatility traders in OTM contracts does not necessarily increase the measurement of VPIN difference. The difference in information asymmetry between option moneyness, is determined by the equilibrium result of informed volatility trader's strategic behavior and the behavior of market makers given the liquidity hedgers' preference and option contracts characteristics. If $\Delta VPINdiff_\tau$ is positive, it can be the result of the following situations: the level of volatility information risk has increased in OTM contracts; the level of volatility information risk has decreased in ATM contracts; both of these scenarios; or the volatility information has increased in both categories but the degree of increase is greater for OTM contracts. All of the above imply that the degree of heterogeneity in information asymmetry across strikes has increased from the last period. Subsequently, we should expect the slope of volatility smile to increase. To further investigate the role of volatility information asymmetry in determining the shape of the smile, I take advantage of the third hypothesis derived from the model. I am going to include an interactive term $VPINdiff_\tau \cdot \Delta\sigma_\tau^{hist}$ in the regression that attempts to capture the sensitivity of skew movement to a change in historical volatility in relation to the varying information structure of options. If historical volatility is affecting the volatility skew for non-information asymmetry reasons, the estimated coefficient on $VPINdiff_\tau \cdot \Delta\sigma_\tau^{hist}$ is expected not to be significantly different from zero.

There certainly are investors who trade options for reasons not included in this model. While potentially there are information traders who use options to exploit their informational advantage on the directional movement of underlying asset, it is not obvious why it would affect market makers and the price of options to the extent that it will bias analysis. Not only such directional risk can be delta-hedged in the underlying market, there are also many alternative vehicles to trade private directional information such as margin trading. However, the option market is the only place where investors can trade volatility information. The presence of informed directional traders will only create more noise to the effort of isolating behaviors of the volatility information trader. On a different note, as shown in previous sections, the behavior of an informed volatility trader concerns market makers in many ways. The bid-ask spread of options is among the first things that are directly affected as a result of volatility information trading. Therefore, it is my view that the documented potential correlation between the shapes of the implied volatility smile and the bid-ask spread in the options market are the simultaneous consequences of volatility information trading in the option market, rather than a causality relationship. Including option spreads into the test will therefore produce misspecification errors and bias the result.

However, it does seem important to control for any liquidity or demand reasons that could potentially affect the IVF. Bollen and Whaley (2004) find that the net demand pressure of options has an impact on the implied volatility function as a result of a 'not perfectly elastic' supply of options. Along a related line, Nordén and Xu (2010) find a positive relationship between the relative liquidity difference in strike prices and the slope of the implied volatility smile. It is possible that a significant relationship between volatility information asymmetry and the skew of IV is driven by liquidity reasons. For example, when there is a large increase in the demand pressure of an option series, certain options may become more expensive depending on the supply elasticity. However, if this increase in options demand is distributed in an imbalanced way across different strikes, it will also affect the estimation of VPINs. Similarly, when the relative liquidity between OTM and ATM contracts varies from time to time, it may affect the relative volatility information risk measures if the change in liquidity is unbalanced between buys and sells (e.g., if OTM option's liquidity has decreased because there are fewer buying orders, this decreases the relative liquidity ratio as well as increases the OTM VPIN; both can lead to higher implied volatility skew). I control for net demand pressure and its effect on IV skew through two terms—Net Demand and Relative Demand. These control variables are constructed by taking the relative difference between buy-initiated orders and sell-initiated orders from non-market makers across all strike prices and calculating the relative difference between the OTM and ATM net demands, respectively. Specifically:

$$ND = \frac{\sum_i Buys - \sum_i Sells}{\sum_i Buys + \sum_i Sells} \tag{39}$$

$$RND = \frac{(\sum_i Buys_{OTM} - \sum_i Sells_{OTM}) - (\sum_i Buys_{ATM} - \sum_i Sells_{ATM})}{\sum_i Buys_{All} + \sum_i Sells_{All}} \quad (40)$$

I control for the relative liquidity between the OTM and ATM contracts through the term RL, which is calculated according to:

$$RL = \frac{Ln(Volume_{OTM}) - Ln(Volume_{ATM})}{Ln(Volume_{OTM} + Volume_{ATM})} \quad (41)$$

5.3 *Data & Sample Selection*

Skew Characteristics

Figure 23 illustrates the average time-series properties of options IV skew, ATM implied volatility, and historical volatility over all the equity options in the data before filtering for regression. A revealing characteristic of the graph is that for most part, the volatility skew shows an inverse relationship with the ATM implied volatility. This implies that when ATM options become less expensive, the implied volatility of OTM options either increases or decreases by an amount no more than the ATM options. In addition, another interesting feature from the figure is that while historical volatility is mostly higher than implied volatility in ATM options, it also displays a much smoother pattern over the sample period.

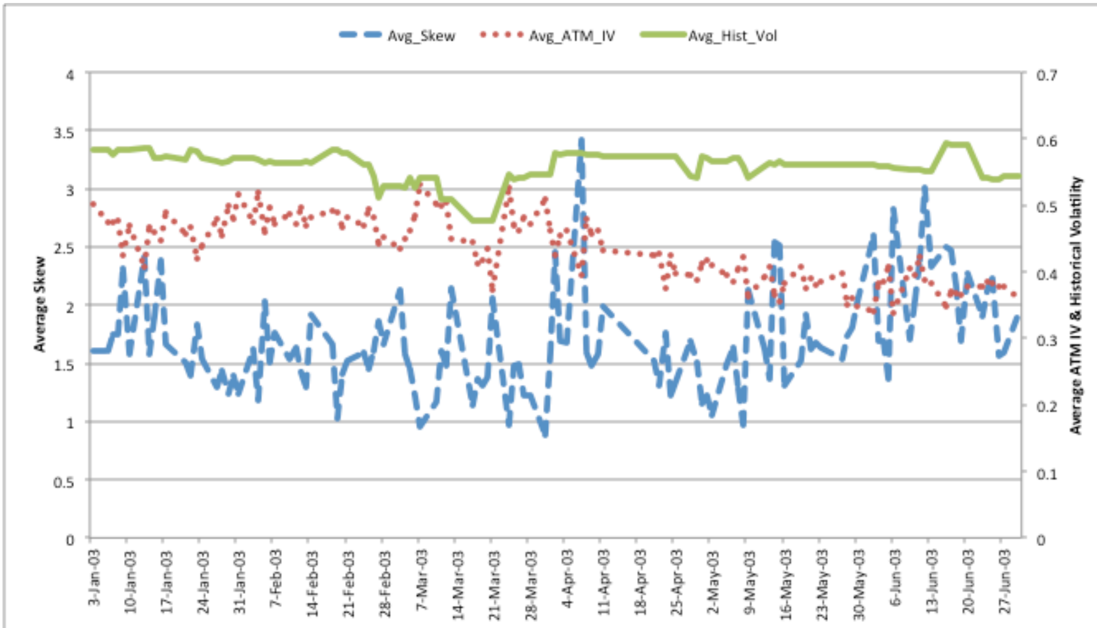

Figure 23. US Equity Options Skew, ATM Implied Volatility, and Historical Volatility from January 2003 to June 2003
The average skew, ATM implied volatility, and Historical volatility is obtained by taking the mean value across all available stocks before filtering the data.

Sample Selection

In order to investigate the role of heterogeneous information asymmetry in determining the implied volatility skew, I will use previously estimated VPINs to proxy for the information risk in each option category. The classification of option categories in the investigation is therefore consistent with the classification in the VPIN estimation. Namely, the At The Money (ATM) group consists of Put options that have moneyness ($\frac{K}{S}$) between 0.9 and 1.05, and anything greater than 10% OTM ($\frac{K}{S} < 0.9$) is classified as in the Out of The Money (OTM) group. The intra-day quotes and trades data required to estimate the VPIN are from CBOE; the details of the data and sample selection regarding VPIN estimation can be found in the earlier section.

While quoting prices during the trading day requires market makers be able to detect useful information from noise, prices at the market opening are more likely to reflect the best ex-ante strategy of market makers after they have digested all the information from the previous trading day. So market opening option prices are obtained from the first available quotes from the CBOE dataset from Jan 2003 to June 2003. Data involving price and other characteristics of underlying stock are from the CRSP database. I obtain the 365-day historical volatility from OptionMetrics. The interest rate is proxied by the rate of 1-year Treasury bills.

When calculating the daily skew in equation (37), the implied volatility of each category is calculated based on the mid-point of the open bid-ask prices of the two contracts with the most extreme strike prices. I will only use the near-term maturity contract with a ‘next-month’ expiration date for each of the two strikes, meaning a maturity time between 35 to 63 days. Moreover, during the implied volatility calculation, the closing price of the underlying stock from the previous trading day is used. Finally, an equal-weighted average on the daily skew is taken over a ten-day interval to obtain the average skew for each period τ.

To qualify as an observation for estimating equation (38), every trading day within period τ must have a valid data point. A trading day is defined to be a valid data point for an option series at day t if: (a) There must be data available to construct the skew variable. (b) A VPIN must be available for both OTM and ATM categories to calculate the VPIN difference. (c) There must be historical volatility data available for the underlying stock. Option trades data during the day are grouped according to the moneyness of the contract at the market opening; all near-term contracts in each category are included when calculating trading volume.

The value of the underlying stock’s return is determined by its cumulative returns over the period τ, whereas for RND and RL, their values are drawn from the 10-day average in each period.

5.4 *Summary Statistics*

Table 8 provides the summary statistics of all the main variables involved in the regression analysis. The mean of the IV skew in the sample is 1.87, which implies that a one percent decrease in the strike price of a Put option (equivalent to saying that the contract is one percent *more* OTM) is on average associated with a 1.87 percent increase in implied volatility. The average change of skew measure is 0.15, indicating that we often see an increase in volatility skew during the sample period. The mean of the VPIN of OTM contracts is 0.15 greater than ATM contracts. The historical volatility of underlying stock has a mean of 0.48, and these equities experienced an average return of 0.02 or 2% over the sample period. The change in volatility has a mean of -0.01. The joint fact of a negative mean and a negatively skewed distribution seems to indicate that most volatility movements during the sample period are associated with the downside. Net demand pressure has a mean of -0.07, and the average relative liquidity ratio is -0.2. These numbers imply that in this sample, the selling pressure in the options market is higher than the purchasing pressure and that average OTM transaction volume is smaller than ATM volume, respectively.

Table 8. Summary Statistics

This table reports summary statistics of the filtered sample from Implied Volatility Skew estimation. IV Skew is calculated using mid-point of open bid-ask prices of the two contracts with most extreme strike prices. VPIN difference ($VPINdiff$) is obtained from OTM VPIN minus ATM VPIN. Historical volatility is the annualized volatility of each underlying stock. The value of underlying stock's return is determined by its cumulative returns over the period τ, whereas for RND and RL, their values are drawn from the 10-day average in each period.

	Mean	Std	Skew	Kurt	Min	Max
IV Skew	1.87	1.54	6.59	74.96	-3.28	18.42
Change in Skew ($\Delta Skew$)	0.15	1.27	5.86	52.76	-2.79	12.16
VPIN difference ($VPINdiff$)	0.15	0.11	-0.51	2.89	-0.21	0.37
Historical Volatility	0.48	0.16	1.25	5.61	0.22	1.25
Change in Volatility ($\Delta\sigma^{hist}$)	-0.01	0.03	-7.40	64.84	-0.27	0.02
Stock Return (SR)	0.02	0.06	1.07	5.70	-0.13	0.24
Net Demand (ND)	-0.07	0.22	-0.19	3.72	-0.74	0.66
Relative Net Demand (RND)	-0.03	0.20	-0.02	3.80	-0.87	0.51
Relative Liquidity Ratio (RL)	-0.20	0.14	0.07	3.42	-0.55	0.25

5.5 *Results*

In order to investigate the effect of volatility information risk and the difference of which across strike prices on the shape and dynamics of implied volatility skew, I use simple OLS to estimate equation (38). The overall results are quite conclusive: the relative information asymmetry appears to play an important role in determining the skewness of implied volatility in the options market. Table 9 presents these results in five columns with variations in control variables. In column (1), the estimated coefficient for change in information risk difference between OTM and ATM contracts ($\Delta VPINdiff$) is positive and significant at 1% level. This is consistent with the theoretical prediction and the hypothesis that a higher discrepancy in volatility information risk between OTM and ATM contracts will lead to a greater implied volatility skewness level. As discussed previously, the impact of volatility information risk (measured by VPIN) differentials across moneyness on option IV skewness could be a result of changes in demand pressure or relative liquidity under certain circumstances. To take these scenarios into consideration when investigating the role of heterogeneity in information asymmetry, different combinations of the three control variables—Net Demand, Relative Net Demand, and Relative Liquidity—are included in the regressions; their results are shown in columns (3), (4), and (5) of Table 9. The only significant estimate is the coefficient for RL, which as expected has a positive sign and is different from zero at 10% significant level. Yet in all three columns, the results show that neither has much affected the economic nor statistical significance of the $\Delta \mathrm{VPINdiff}$ coefficient.

Table 9. Volatility Information Risk and IV Skew

	(1) ΔSkew	(2) ΔSkew	(3) ΔSkew	(4) ΔSkew	(5) ΔSkew
$\Delta\sigma^{hist}$	-34.31**	-34.39**	-34.35**	-33.48**	-33.63**
	(-11.95)	(-12.00)	(-12.01)	(-11.61)	(-11.68)
$\mathrm{VPINdiff} \cdot \Delta\sigma^{hist}$	219.4**	219.9**	222.3**	216.6**	219.4**
	(8.005)	(8.039)	(8.122)	(7.936)	(8.045)
ΔVPINdiff	3.500**	3.316**	3.561**	3.196**	3.267**
	(3.164)	(2.980)	(3.229)	(2.868)	(2.909)
SR		1.634			0.831
		(1.335)			(0.657)
ND			0.469		0.481
			(1.540)		(1.420)
RND				-0.268	-0.0226
				(-0.767)	(-0.0595)
RL				0.853*	0.920*
				(1.787)	(1.869)
Constant	0.0809	0.0401	0.118	0.240*	0.277*
	(1.097)	(0.503)	(1.527)	(2.003)	(1.987)
Observations	185	185	185	185	185
R-squared	0.460	0.465	0.467	0.470	0.481

This table reports the regression results of average implied volatility skew from January 2003 to June 2003. The dependent variable is average implied volatility skew of selected stocks quoted on the CBOE over an interval of 10 days. $\Delta\sigma^{hist}$ is the annualized historical volatility of each underlying stock. ΔVPINdiff is the change of difference in VPIN, which is calculated using OTM VPIN minus ATM VPIN of the same underlying stock. SR is the cumulative stock return over the same interval. ND is the average Net Demand by non-market makers. RND and RL are estimates of Relative Net Demand and Relative Liquidity between OTM contracts and ATM contracts. During the sample period. T-statistics are shown in parentheses; the significance level of coefficients is shown according to criteria: ** p<0.01, * p<0.1.

Another insight from this dissertation is that the non-symmetric structure of information asymmetry between OTM and ATM contracts plays a critical role in deciding the degree to which a change in volatility uncertainty affects the shape of the implied volatility smile. To explore this hypothesis, an interactive term that measures the difference in VPIN between the OTM and ATM contracts, $VPINdiff$, is included in all regressions. I find that regardless of the control variables in use, columns (1) to (5) unanimously demonstrate a positive and highly significant estimate for the coefficient of $\mathrm{VPINdiff} \cdot \Delta\sigma^{hist}$, providing strong evidence in favor of the hypothesis. Moreover, it is very interesting to jointly interpret the estimates for $\Delta\sigma^{hist}$ and

$VPINdiff \cdot \Delta\sigma^{hist}$. For example, if we look at the results in column (5), the estimate for $\Delta\sigma^{hist}$ and $VPINdiff \cdot \Delta\sigma^{hist}$ is -33.63 and 219.4, respectively. This implies that if the historical volatility of a stock increases and the level of information asymmetry is identical across strike prices, then the slope of the IV smile will actually decrease. However, if we pick the average stock in the sample, which has $VPINdiff$ of 0.15 (i.e., OTM contracts have more volatility information risk than ATM contracts), then the net effect on the IV skew approximately equals 33.63+0.15*219.4=-0.72—very close to zero in net effect. If we select another stock whose excess information risk in OTM contract verses ATM contract is greater than average, an increase in historical volatility may actually increase the implied volatility skew in the options market. These results suggest that the structure of information asymmetry not only affects how changes in volatility uncertainty affect implied volatility skew, it even has the ability to change the direction of this effect should the volatility information risk be significantly higher in OTM contracts than ATM contracts. These findings are also consistent with the intuition behind the theoretical framework. Recall that in equation (17) from Chapter 2 that the equilibrium option price is the expected value of options, given their prices in either volatility states. An increase of volatility uncertainty increases the option values in both states. However, in addition to the non-linearity between option value and volatility in the conditional Black-Scholes formula, a higher information risk in OTM also implies that a larger portion of increase in option value from the High volatility state is carried into the present price than the portion from the Low volatility state.[29] As a result, it is possible that given a large information risk differential, an increase in

[29] Of course, these statements are made under the assumptions that volatility uncertainty is a linear function of historical volatility and the conditional B-S formula is the correct option pricing formula after true volatility is revealed.

historical volatility may actually increase the implied volatility skew, as shown in Figure 14.

CHAPTER 6 Conclusion

In this dissertation, I develop a new model using an asymmetric information framework to study the behavior of volatility information traders in option markets. The theory confirms that leverage, liquidity, and transaction cost are important factors that influence informed investor's investment strategies. By allowing traders to simultaneously select multiple option contracts, I show that as they attempt to balance the tradeoffs between various factors, OTM option contracts will contain higher information risk as an equilibrium outcome.

I also carefully establish a relationship between volatility information trading and option prices. I show that the potential heterogeneity in information asymmetry across option strikes is an important contributor to the observed spread structure in option markets and that OTM contracts tend to have larger spreads. In addition, I expand the current understanding of the role of information asymmetry in the implied volatility skew by making a connection with the behavior of volatility traders. Because the degree of heterogeneity in information risk within the same option series may vary from case to case as market condition changes, I show that a higher difference in information risk across strikes will result in a greater slope of implied volatility skew.

In addition to providing a theoretical foundation for existing empirical studies, I also include a number of tests on the implication of the theoretical model developed in this dissertation. Using several datasets, including two proprietary ones from the Chicago Board Option Exchange (CBOE), I am able to estimate the VPIN variable for the US equity option market, and provide evidence that OTM option contracts on average, have a higher probability of information trading than ATM option contracts. I also show that volatility information risk accounts for a considerable proportion in explaining the empirical spread structure in the US equity option market. Furthermore, in the last chapter, I provide new evidence that differences in information asymmetry within the same option series not only help to directly explain the dynamics of implied volatility skew; they also play a central role in determining the degree to which changes in volatility affect the dynamics of implied volatility skew.

Finally, I believe it will be very interesting for future research to improve and expand the content of this dissertation in a number of ways. I would love to explore an extension of the theory for a different or more complex economy. Doing so could generate some fascinating implications that would allow us to empirically study and compare the effect of volatility information trading on other types of option markets, from interest rates to foreign exchanges. It would also be beneficial to the understanding of this literature if the idea of the theoretical model could be applied in a dynamic setting, which would help address numerous questions that this static model is not able to systematically answer.

Appendix

Notations in Appendix: $C_i(\sigma) = C_{t=1}(\sigma, k_i)$

<u>Solve for equilibrium:</u>

$$\max_{\rho_i,\rho_j} \frac{w\rho_i}{A_i}\left(C_i(\overline{\sigma}) - A_i\right) + \frac{w\rho_j}{A_j}\left(C_j(\overline{\sigma}) - A_j\right)$$

$$s.t. \quad \rho_i + \rho_j = 1$$

$$\Rightarrow \; Max_{\rho_i} \; \frac{w\rho_i \cdot \left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)(1-\alpha)h_i}{\alpha\rho_i + 2(1-\alpha)h_i} \cdot \frac{1}{A_i} + \frac{w\rho_j \cdot \left(C_j(\overline{\sigma}) - C_j(\underline{\sigma})\right)(1-\alpha)h_j}{\alpha\rho_j + 2(1-\alpha)h_j} \cdot \frac{1}{A_j}$$

$$s.t. \quad \rho_i + \rho_j = 1$$

$$\Rightarrow \; Max_{\rho_i} \; \frac{w\rho_i \cdot \left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)(1-\alpha)h_i}{\alpha\rho_i C_i(\overline{\sigma}) + (1-\alpha)h_i(C_i(\overline{\sigma}) + C_i(\underline{\sigma}))} + \frac{w\rho_j \cdot \left(C_j(\overline{\sigma}) - C_j(\underline{\sigma})\right)(1-\alpha)h_j}{\alpha\rho_j C_j(\overline{\sigma}) + (1-\alpha)h_j(C_j(\overline{\sigma}) + C_j(\underline{\sigma}))}$$

$$s.t. \quad \rho_i + \rho_j = 1$$

FOC:

$$\rho_i: \frac{w\left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)(1-\alpha)h_i \cdot \left(\alpha\rho_i C_i(\overline{\sigma}) + (1-\alpha)h_i(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right) - w\rho_i\left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)(1-\alpha)h_i \cdot \alpha C_i(\overline{\sigma})}{\left[\alpha\rho_i C_i(\overline{\sigma}) + (1-\alpha)h_i\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right]^2}$$

$$= \lambda$$

$$\frac{w\left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)(1-\alpha)^2 h_i^2\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)}{\left[\alpha\rho_i C_i(\overline{\sigma}) + (1-\alpha)h_i\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right]^2} = \lambda$$

λ: $\rho_i + \rho_j = 1$

So

$$\frac{h_i^2\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)}{\left[\alpha\rho_i C_i(\overline{\sigma}) + (1-\alpha)h_i\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right]^2} = \frac{h_j^2\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right)\left(C_j(\overline{\sigma}) - C_j(\underline{\sigma})\right)}{\left[\alpha\rho_j C_j(\overline{\sigma}) + (1-\alpha)h_j\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right)\right]^2}$$

And solve this:

$$\frac{h_i\sqrt{\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)}}{\alpha\rho_i C_i(\overline{\sigma}) + (1-\alpha)h_i\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)} = \frac{h_j\sqrt{\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right)\left(C_j(\overline{\sigma}) - C_j(\underline{\sigma})\right)}}{\alpha\rho_j C_j(\overline{\sigma}) + (1-\alpha)h_j\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right)}$$

$$h_i\sqrt{\left(C_i(\overline{\sigma})^2 - C_i(\underline{\sigma})^2\right)}\left[\alpha(1-\rho_i)C_j(\overline{\sigma}) + (1-\alpha)h_j\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right)\right]$$

$$= h_j\sqrt{\left(C_j(\overline{\sigma})^2 - C_j(\underline{\sigma})^2\right)}\left[\alpha\rho_i C_i(\overline{\sigma}) + (1-\alpha)h_i\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right]$$

$$h_i\sqrt{\left(C_i(\overline{\sigma})^2 - C_i\left(\underline{\sigma}\right)^2\right)}\left[\alpha C_j(\overline{\sigma}) + (1-\alpha)h_j\left(C_j(\overline{\sigma}) + C_j\left(\underline{\sigma}\right)\right)\right] - h_j\sqrt{\left(C_j(\overline{\sigma})^2 - C_j\left(\underline{\sigma}\right)^2\right)}\ (1-\alpha)h_i\left(C_i(\overline{\sigma}) + C_i\left(\underline{\sigma}\right)\right)$$

$$= h_j\sqrt{\left(C_j(\overline{\sigma})^2 - C_j\left(\underline{\sigma}\right)^2\right)}\ \ \alpha\rho_i C_i(\overline{\sigma}) + h_i\sqrt{\left(C_i(\overline{\sigma})^2 - C_i\left(\underline{\sigma}\right)^2\right)}\ \ \alpha\rho_i C_j(\overline{\sigma})$$

$$\rho_i\left(h_j\sqrt{\left(C_j(\overline{\sigma})^2 - C_j\left(\underline{\sigma}\right)^2\right)}\,\alpha C_i(\overline{\sigma}) + h_i\sqrt{\left(C_i(\overline{\sigma})^2 - C_i\left(\underline{\sigma}\right)^2\right)}\ \alpha C_j(\overline{\sigma})\right)$$

$$= \ \alpha C_j(\overline{\sigma})h_i\sqrt{\left(C_i(\overline{\sigma})^2 - C_i\left(\underline{\sigma}\right)^2\right)}$$

$$+ (1-\alpha)h_i h_j\left[\sqrt{\left(C_i(\overline{\sigma})^2 - C_i\left(\underline{\sigma}\right)^2\right)}\left(C_j(\overline{\sigma}) + C_j\left(\underline{\sigma}\right)\right) - \sqrt{\left(C_j(\overline{\sigma})^2 - C_j\left(\underline{\sigma}\right)^2\right)}\left(C_i(\overline{\sigma}) + C_i\left(\underline{\sigma}\right)\right)\right]$$

So,

$$\rho_i^* = \frac{\alpha h_i C_j(\overline{\sigma})\sqrt{\left(C_i(\overline{\sigma})^2 - C_i\left(\underline{\sigma}\right)^2\right)} + (1-\alpha)h_i h_j\left[\sqrt{\left(C_i(\overline{\sigma})^2 - C_i\left(\underline{\sigma}\right)^2\right)}\left(C_j(\overline{\sigma}) + C_j\left(\underline{\sigma}\right)\right) - \sqrt{\left(C_j(\overline{\sigma})^2 - C_j\left(\underline{\sigma}\right)^2\right)}\left(C_i(\overline{\sigma}) + C_i\left(\underline{\sigma}\right)\right)\right]}{\alpha h_i C_j(\overline{\sigma})\sqrt{\left(C_i(\overline{\sigma})^2 - C_i\left(\underline{\sigma}\right)^2\right)} + \alpha h_j C_i(\overline{\sigma})\sqrt{\left(C_j(\overline{\sigma})^2 - C_j\left(\underline{\sigma}\right)^2\right)}}$$

Proof for Proposition 1.1

Current assumptions:

$$K_i > K_j \geq S, \qquad \frac{C_i(\overline{\sigma}) - C_i(\underline{\sigma})}{C_i(\underline{\sigma})} > \frac{C_j(\overline{\sigma}) - C_j(\underline{\sigma})}{C_j(\underline{\sigma})}$$

$$C_i(\quad) < C_j(\quad), and\ \frac{\partial C_i(\quad)}{\partial \sigma} < \frac{\partial C_j(\quad)}{\partial \sigma}$$

Proof by contradiction, suppose:

$$\sqrt{\left(C_i(\overline{\sigma})^2 - C_i(\underline{\sigma})^2\right)}\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right) - \sqrt{\left(C_j(\overline{\sigma})^2 - C_j(\underline{\sigma})^2\right)}\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right) \geq 0$$

$$\sqrt{\frac{\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)}{\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right)\left(C_j(\overline{\sigma}) - C_j(\underline{\sigma})\right)}} \leq \frac{\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)}{\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right)}$$

$$\sqrt{\frac{\left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)}{\left(C_j(\overline{\sigma}) - C_j(\underline{\sigma})\right)}} \leq \sqrt{\frac{\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)}{\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right)}}$$

$$\frac{\left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)}{\left(C_j(\overline{\sigma}) - C_j(\underline{\sigma})\right)} \leq \frac{\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)}{\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right)}$$

$$\left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right) \leq \left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\left(C_j(\overline{\sigma}) - C_j(\underline{\sigma})\right)$$

$$C_i(\overline{\sigma}) \cdot C_j(\underline{\sigma}) \leq C_i(\underline{\sigma}) \cdot C_j(\overline{\sigma})$$

$$\frac{C_i(\overline{\sigma}) - C_i(\underline{\sigma})}{C_i(\underline{\sigma})} \leq \frac{C_j(\overline{\sigma}) - C_j(\underline{\sigma})}{C_j(\underline{\sigma})} \boxtimes$$

Proof for Proposition 1.2

Informed investor will invest everything into i-contract if $\rho_i^* \geq 1$

$$(1-\alpha)h_i h_j\left[\sqrt{C_i(\overline{\sigma})^2 - C_i(\underline{\sigma})^2}\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right) - \sqrt{C_j(\overline{\sigma})^2 - C_j(\underline{\sigma})^2}\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right]$$

$$> \alpha h_j C_i(\overline{\sigma})\sqrt{C_j(\overline{\sigma})^2 - C_j(\underline{\sigma})^2}$$

$$\frac{(1-\alpha)h_i}{\alpha} > \frac{C_i(\overline{\sigma})\sqrt{C_j(\overline{\sigma})^2 - C_j(\underline{\sigma})^2}}{\left[\sqrt{C_i(\overline{\sigma})^2 - C_i(\underline{\sigma})^2}\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right) - \sqrt{C_j(\overline{\sigma})^2 - C_j(\underline{\sigma})^2}\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right]}$$

Proof for Proposition 1.3 and 1.4

Comparative Statics:

$$let\ D_i = \left[\sqrt{C_i(\overline{\sigma})^2 - C_i(\underline{\sigma})^2}\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right) - \sqrt{C_j(\overline{\sigma})^2 - C_j(\underline{\sigma})^2}\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right] > 0,$$

$$\Sigma_i = \sqrt{\left(C_i(\overline{\sigma})^2 - C_i(\underline{\sigma})^2\right)} > 0$$

recall

$$\rho_i^* = \frac{\alpha h_i C_j(\overline{\sigma})\sqrt{\left(C_i(\overline{\sigma})^2 - C_i(\underline{\sigma})^2\right)} + (1-\alpha)h_i h_j\left[\sqrt{\left(C_i(\overline{\sigma})^2 - C_i(\underline{\sigma})^2\right)}\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right) - \sqrt{\left(C_j(\overline{\sigma})^2 - C_j(\underline{\sigma})^2\right)}\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right]}{\alpha h_i C_j(\overline{\sigma})\sqrt{\left(C_i(\overline{\sigma})^2 - C_i(\underline{\sigma})^2\right)} + \alpha h_j C_i(\overline{\sigma})\sqrt{\left(C_j(\overline{\sigma})^2 - C_j(\underline{\sigma})^2\right)}}$$

$$\rho_i^* = \frac{\alpha h_i C_j(\overline{\sigma})\Sigma_i + (1-\alpha)h_i h_j D_i}{\alpha h_i C_j(\overline{\sigma})\Sigma_i + \alpha h_j C_i(\overline{\sigma})\Sigma_j}$$

then,

$$\frac{\partial \rho_i^*}{\partial \alpha} = \frac{\left(h_i C_j(\overline{\sigma})\Sigma_i - h_i h_j D_i\right)\left(\alpha h_i C_j(\overline{\sigma})\Sigma_i + \alpha h_j C_i(\overline{\sigma})\Sigma_j\right) - \left(\alpha h_i C_j(\overline{\sigma})\Sigma_i + (1-\alpha)h_i h_j D_i\right)\left(h_i C_j(\overline{\sigma})\Sigma_i + h_j C_i(\overline{\sigma})\Sigma_j\right)}{\alpha h_i C_j(\overline{\sigma})\Sigma_i + \alpha h_j C_i(\overline{\sigma})\Sigma_j}$$

$$= \frac{-h_i h_j D_i \alpha\left(h_i C_j(\overline{\sigma})\Sigma_i - h_j C_i(\overline{\sigma})\Sigma_j\right) - (1-\alpha)h_i h_j D_i\left(h_i C_j(\overline{\sigma})\Sigma_i + h_j C_i(\overline{\sigma})\Sigma_j\right)}{\alpha h_i C_j(\overline{\sigma})\Sigma_i + \alpha h_j C_i(\overline{\sigma})\Sigma_j}$$

$$\boldsymbol{\frac{\partial \rho_i^*}{\partial \alpha} = \frac{-h_i h_j D_i\left(h_i \overline{C}_j \Sigma_i + h_j \overline{C}_i \Sigma_j\right)}{\left(\alpha h_i \overline{C}_j \Sigma_i + \alpha h_j \overline{C}_i \Sigma_j\right)^2} < 0}$$

$$\frac{d\rho_i^*}{dh_i} = \frac{\partial \rho_i^*}{\partial h_i} + \frac{\partial h_j}{\partial h_i}\cdot\frac{\partial \rho_i^*}{\partial h_j}$$

Since

$$\frac{\partial h_i}{\partial h_j} < 0, \frac{\partial h_j}{\partial h_i} < 0, \quad and$$

$$\frac{\partial \rho_i^*}{\partial h_i} = \frac{\left(\alpha C_j(\overline{\sigma})\Sigma_i + (1-\alpha)h_j D_i\right)\left(\alpha h_i C_j(\overline{\sigma})\Sigma_i + \alpha h_j C_i(\overline{\sigma})\Sigma_j\right) - \left(\alpha h_i C_j(\overline{\sigma})\Sigma_i + (1-\alpha)h_i h_j D_i\right)\alpha C_j(\overline{\sigma})\Sigma_i}{\left[\alpha h_i C_j(\overline{\sigma})\Sigma_i + \alpha h_j C_i(\overline{\sigma})\Sigma_j\right]^2}$$

$$\frac{\partial \rho_i^*}{\partial h_i} = \frac{\alpha^2 C_i(\overline{\sigma})C_j(\overline{\sigma})\Sigma_i\Sigma_j D_i h_j + a(1-\alpha)h_j^2 C_i(\overline{\sigma})\Sigma_j D_i}{\left[\alpha h_i C_j(\overline{\sigma})\Sigma_i + \alpha h_j C_i(\overline{\sigma})\Sigma_j\right]^2} > 0$$

$$\frac{\partial \rho_i^*}{\partial h_j} = \frac{(1-\alpha)h_i D_i\left(\alpha h_i C_j(\overline{\sigma})\Sigma_i + \alpha h_j C_i(\overline{\sigma})\Sigma_j\right) - \left(\alpha h_i C_j(\overline{\sigma})\Sigma_i + (1-\alpha)h_i h_j D_i\right)\alpha C_i(\overline{\sigma})\Sigma_j}{\left[\alpha h_i C_j(\overline{\sigma})\Sigma_i + \alpha h_j C_i(\overline{\sigma})\Sigma_j\right]^2}$$

$$\frac{\partial \rho_i^*}{\partial h_j} = \frac{a(1-\alpha)h_i^2 C_j(\overline{\sigma})\Sigma_i D_i - \alpha^2 C_i(\overline{\sigma})C_j(\overline{\sigma})\Sigma_i\Sigma_j h_i}{\left[\alpha h_i C_j(\overline{\sigma})\Sigma_i + \alpha h_j C_i(\overline{\sigma})\Sigma_j\right]^2}$$

$$\frac{\partial h_j}{\partial h_i} = -1$$

So

$$\frac{d\rho_i^*}{dh_i} = \frac{\partial \rho_i^*}{\partial h_i} + \frac{\partial h_j}{\partial h_i} \cdot \frac{\partial \rho_i^*}{\partial h_j}$$

$$= \frac{\alpha^2 C_i(\overline{\sigma})C_j(\overline{\sigma})\Sigma_i\Sigma_j\left(D_i h_j + h_i\right) + a(1-\alpha)D_i\left(h_j^2 C_i(\overline{\sigma})\Sigma_j - h_i^2 C_j(\overline{\sigma})\Sigma_i\right)}{\left[\alpha h_i C_j(\overline{\sigma})\Sigma_i + \alpha h_j C_i(\overline{\sigma})\Sigma_j\right]^2}$$

$$= \frac{\alpha D_i h_i C_j(\overline{\sigma})\Sigma_i\left(\alpha C_i(\overline{\sigma})\Sigma_j - (1-\alpha)h_i\right) + \alpha^2 C_i(\overline{\sigma})C_j(\overline{\sigma})\Sigma_i\Sigma_j D_i h_j + a(1-\alpha)D_i h_j^2 C_i(\overline{\sigma})\Sigma_j}{\left[\alpha h_i C_j(\overline{\sigma})\Sigma_i + \alpha h_j C_i(\overline{\sigma})\Sigma_j\right]^2}$$

Because $\quad \left[\alpha h_i C_j(\overline{\sigma})\Sigma_i + \alpha h_j C_i(\overline{\sigma})\Sigma_j\right]^2 > 0$,

$$\frac{d\rho_i^*}{dh_i} > 0 \ iff \ \alpha^2 C_i(\overline{\sigma})C_j(\overline{\sigma})\Sigma_i\Sigma_j\left(Dh_j + h_i\right) + a(1-\alpha)D_i\left(h_j^2 C_i(\overline{\sigma})\Sigma_j - h_i^2 C_j(\overline{\sigma})\Sigma_i\right) > 0$$

Recall that when $0 < \rho_i^* < 1$, *we have:* $\frac{(1-\alpha)h_i}{\alpha} < \frac{C_i(\overline{\sigma})\Sigma_j}{D_i}$

Proof by contradiction:

Suppose $\quad \alpha^2 C_i(\overline{\sigma})C_j(\overline{\sigma})\Sigma_i\Sigma_j\left(D_i h_j + h_i\right) + a(1-\alpha)D_i\left(h_j^2 C_i(\overline{\sigma})\Sigma_j - h_i^2 C_j(\overline{\sigma})\Sigma_i\right) < 0$

$$\alpha^2 C_i(\overline{\sigma})C_j(\overline{\sigma})\Sigma_i\Sigma_j\left(D_i h_j + h_i\right) < a(1-\alpha)D_i h_i^2 C_j(\overline{\sigma})\Sigma_i$$

$$aC_i(\overline{\sigma})\Sigma_j\left(D_i h_j + h_i\right) < (1-\alpha)D_i h_i^2$$

$$aC_i(\overline{\sigma})\Sigma_j h_i < (1-\alpha)D_i h_i^2$$

$$\frac{C_i(\overline{\sigma})\Sigma_j}{D_i} < \frac{(1-\alpha)h_i}{\alpha} \boxtimes$$

<u>Proof for Proposition 1.5</u>

Claim: In equilibrium, $PIN_1^{Vol} \geq PIN_2^{Vol}$

Proof by Contradiction:

Current assumption: $\frac{C_i(\overline{\sigma})-C_i(\underline{\sigma})}{C_i(\underline{\sigma})} \geq \frac{C_j(\overline{\sigma})-C_j(\underline{\sigma})}{C_j(\underline{\sigma})}$ (1)

We know at equilibrium, marginal profit from each contract should be the same:

$$\frac{(C_i(\overline{\sigma})-A_i)}{A_i} = \frac{\left(C_j(\overline{\sigma})-A_j\right)}{A_j} \quad (2)$$

Because the probabilities add up to 1:

$$\frac{\alpha\rho_i^*}{\alpha\rho_i^*+2(1-\alpha)h_i} + \frac{2(1-\alpha)h_i}{\alpha\rho_i^*+2(1-\alpha)h_i} = 1\ \forall i \ (3)$$

Suppose $PIN_1^{Vol} < PIN_2^{Vol}$

Then, by (3),

$$\frac{(1-\alpha)h_1}{\alpha\rho_1^*+2(1-\alpha)h_1} > \frac{(1-\alpha)h_2}{\alpha\rho_2^*+2(1-\alpha)h_2} \quad (4)$$

Because the equilibrium price is expressed as:

$A_i = \frac{\alpha\rho_i^*+(1-\alpha)h_i}{\alpha\rho_i^*+2(1-\alpha)h_i} * C_i(\overline{\sigma}) + \frac{(1-\alpha)h_i}{\alpha\rho_i^*+2(1-\alpha)h_i} * C_i(\underline{\sigma})$

$$\frac{A_i}{C_i(\overline{\sigma})} = \frac{\alpha\rho_i^*+(1-\alpha)h_i}{\alpha\rho_i^*+2(1-\alpha)h_i} + \frac{(1-\alpha)h_i}{\alpha\rho_i^*+2(1-\alpha)h_i} * \frac{C_i(\underline{\sigma})}{C_i(\overline{\sigma})}$$

$$= P_i + Q_i\Gamma_i$$

$$\frac{A_1}{C_1(\overline{\sigma})} - \frac{A_2}{C_2(\overline{\sigma})} = P_1 + Q_1 * \Gamma_1 - (P_2 + Q_2 * \Gamma_2)$$

$$= 1 - Q_1 + Q_1 * \Gamma_1 - (1 - Q_2) - Q_2 * \Gamma_2$$

$$= -(Q_1 - Q_2) + Q_1\Gamma_1 - Q_2\Gamma_2 + Q_1\Gamma_2 - Q_1\Gamma_2$$

$$= -(Q_1 - Q_2) + Q_1(\Gamma_1 - \Gamma_2) + \Gamma_2(Q_1 - Q_2)$$

by (1) and (4), $0 < \Gamma_1 < \Gamma_2 < 1$ and $Q_1 > Q_2$

$$\therefore \ \frac{A_1}{C_1(\overline{\sigma})} - \frac{A_2}{C_2(\overline{\sigma})} < 0$$

$$\Rightarrow \frac{(C_1(\overline{\sigma})-A_1)}{A_1} < \frac{(C_2(\overline{\sigma})-A_2)}{A_2} \quad \boxtimes\ (2)$$

Proof for Proposition 2.1:

$$A_i = \frac{\alpha\rho_i^* + (1-\alpha)h_i}{\alpha\rho_i^* + 2(1-\alpha)h_i} * C_i(\overline{\sigma}) + \frac{(1-\alpha)h_i}{\alpha\rho_i^* + 2(1-\alpha)h_i} * C_i(\underline{\sigma})$$

Take total differentiation:

$$\frac{dA_i^*}{d\alpha} = \frac{\partial A_i^*}{\partial\alpha} + \frac{\partial A_i^*}{\partial\rho_i^*}\cdot\frac{\partial\rho_i^*}{\partial\alpha}$$

Each component can be obtained individually:

$$\frac{\partial A_i^*}{\partial\alpha} = \frac{\left[\rho_i^* C_i(\overline{\sigma}) - h_i\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right](\alpha\rho_i^* + 2(1-\alpha)h_i) - (\rho_i^* - 2h_i)\left[\alpha\rho_i^* C_i(\overline{\sigma}) + (1-\alpha)h_i\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right]}{\left(\alpha\rho_i^* + 2(1-\alpha)h_i\right)^2}$$

$$\frac{\partial A_i^*}{\partial\alpha} = \frac{\rho_i^* h_i\left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)}{\left(\alpha\rho_i^* + 2(1-\alpha)h_i\right)^2} > \mathbf{0}$$

$$\frac{\partial A_i^*}{\partial\rho_i^*} = \frac{\alpha C_i(\overline{\sigma})(\alpha\rho_i^* + 2(1-\alpha)h_i) - \left[\alpha\rho_i^* + (1-\alpha)h_i\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right]\alpha}{\left(\alpha\rho_i^* + 2(1-\alpha)h_i\right)^2}$$

$$\frac{\partial A_i^*}{\partial\rho_i^*} = \frac{\alpha(1-\alpha)h_i\left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)}{\left(\alpha\rho_i^* + 2(1-\alpha)h_i\right)^2} > \mathbf{0}$$

$$\frac{\partial\rho_i^*}{\partial\alpha} = \frac{-h_i h_j D_i\left(h_i\overline{C}_j\Sigma_i + h_j\overline{C}_i\Sigma_j\right)}{\left(\alpha h_i\overline{C}_j\Sigma_i + \alpha h_j\overline{C}_i\Sigma_j\right)^2}\begin{cases} < \mathbf{0}\ \boldsymbol{if}\ \boldsymbol{i = 1} \\ > \mathbf{0}\ \boldsymbol{if}\ \boldsymbol{i = 2}\end{cases}$$

First, as discussed previously, the price of option Contract-2 will increase unambiguously:

$$Since \qquad \frac{\partial A_2^*}{\partial\alpha} > 0, \frac{\partial A_2^*}{\partial\rho_1^*} < 0, and\ \frac{\partial\rho_1^*}{\partial\alpha} < 0$$

$$\frac{dA_2^*}{d\alpha} = \frac{\partial A_2^*}{\partial\alpha} + \frac{\partial A_2^*}{\partial\rho_2^*}\cdot\frac{\partial\rho_2^*}{\partial\alpha} > 0$$

Secondly, we need to combine each term to determine the net effect for Contract-1:

$$\frac{dA_1^*}{d\alpha} = \frac{\rho_1^* h_1\left(C_1(\overline{\sigma}) - C_1(\underline{\sigma})\right)}{(\alpha\rho_1^* + 2(1-\alpha)h_1)^2} + \frac{\alpha(1-\alpha)h_1\left(C_1(\overline{\sigma}) - C_1(\underline{\sigma})\right)}{(\alpha\rho_1^* + 2(1-\alpha)h_1)^2}\cdot\frac{-h_1h_2D\left(h_1\overline{C}_2\Sigma_1 + h_2\overline{C}_1\Sigma_2\right)}{\left(\alpha h_1\overline{C}_2\Sigma_1 + \alpha h_2\overline{C}_1\Sigma_2\right)^2}$$

$$= \frac{h_1\left(C_1(\overline{\sigma}) - C_1(\underline{\sigma})\right)}{(\alpha\rho_1^* + 2(1-\alpha)h_1)^2}\left[\rho_1^* - \frac{\alpha(1-\alpha)h_1h_2D_i\left(h_1\overline{C}_2\Sigma_1 + h_2\overline{C}_1\Sigma_2\right)}{\left(\alpha h_1\overline{C}_2\Sigma_1 + \alpha h_2\overline{C}_1\Sigma_2\right)^2}\right]$$

$$= \frac{h_1\left(C_1(\overline{\sigma}) - C_1(\underline{\sigma})\right)}{(\alpha\rho_1^* + 2(1-\alpha)h_1)^2}\left[\frac{\alpha h_1\overline{C}_2\Sigma_1 + (1-\alpha)h_1h_2D_i}{\alpha h_1\overline{C}_2\Sigma_1 + \alpha h_2\overline{C}_1\Sigma_2} - \frac{(1-\alpha)h_1h_2D_i}{\alpha\left(\alpha h_1\overline{C}_2\Sigma_1 + \alpha h_2\overline{C}_1\Sigma_2\right)}\right]$$

$$= \frac{h_1\left(C_1(\overline{\sigma}) - C_1(\underline{\sigma})\right)}{(\alpha\rho_1^* + 2(1-\alpha)h_1)^2} \cdot \frac{\alpha h_1 \overline{C}_2 \Sigma_1}{\alpha h_1 \overline{C}_2 \Sigma_1 + \alpha h_2 \overline{C}_1 \Sigma_2} > \mathbf{0}$$

<u>The relationship between PIN and $\boldsymbol{pr_i(\overline{\sigma}|BUY)}$:</u>

$$pr_i(\overline{\sigma}|BUY) + pr_i(\underline{\sigma}|BUY) = 1 \; where \;\; pr_i(\overline{\sigma}|BUY) = \frac{\alpha\rho_i + (1-\alpha)h_i}{\alpha\rho_i + 2(1-\alpha)h_i}$$

$$pr_1(\overline{\sigma}|BUY) = pr_2(\overline{\sigma}|BUY) \;\Rightarrow\; \frac{(1-\alpha)h_1}{\alpha\rho_1 + 2(1-\alpha)h_1} = \frac{(1-\alpha)h_2}{\alpha\rho_2 + 2(1-\alpha)h_2} \Rightarrow \frac{\alpha\rho_1}{\alpha\rho_1 + 2(1-\alpha)h_1}$$

$$= \frac{\alpha\rho_2}{\alpha\rho_2 + 2(1-\alpha)h_2} \Rightarrow PIN_1 = PIN_2$$

$$PIN_1 = PIN_2 \;\Rightarrow \frac{2(1-\alpha)h_1}{\alpha\rho_1 + 2(1-\alpha)h_1} = \frac{2(1-\alpha)h_2}{\alpha\rho_2 + 2(1-\alpha)h_2} \Rightarrow \frac{\alpha\rho_1 + (1-\alpha)h_1}{\alpha\rho_1 + 2(1-\alpha)h_1}$$

$$= \frac{\alpha\rho_2 + (1-\alpha)h_2}{\alpha\rho_2 + 2(1-\alpha)h_2} \Rightarrow pr_1(\overline{\sigma}|BUY) = pr_2(\overline{\sigma}|BUY)$$

Solve for Bid-price:

$$Max_{\rho_i}\ \frac{w\rho_i}{B_i}\left(B_i - C_i(\underline{\sigma})\right) + \frac{w\rho_j}{B_j}\left(B_j - C_j(\underline{\sigma})\right)$$

$$s.t.\ \ \rho_i + \rho_j = 1$$

$$\Rightarrow\ Max_{\rho_i}\ \frac{w\rho_i \cdot \left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)(1-\alpha)h_i}{\alpha\rho_i + 2(1-\alpha)h_i} \cdot \frac{1}{B_i} + \frac{w\rho_j \cdot \left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)(1-\alpha)h_j}{\alpha\rho_j + 2(1-\alpha)h_j} \cdot \frac{1}{B_j}$$

$$s.t.\ \ \rho_i + \rho_j = 1$$

$$\Rightarrow\ Max_{\rho_i}\ \frac{w\rho_i \cdot \left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)(1-\alpha)h_i}{\alpha\rho_i C_i(\underline{\sigma}) + (1-\alpha)h_i(C_i(\overline{\sigma}) + C_i(\underline{\sigma}))} + \frac{w\rho_j \cdot \left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)(1-\alpha)h_j}{\alpha\rho_j C_j(\underline{\sigma}) + (1-\alpha)h_j(C_j(\overline{\sigma}) + C_j(\underline{\sigma}))}$$

$$s.t.\ \ \rho_i + \rho_j = 1$$

FOC:

$$\rho_i: \frac{w\left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)(1-\alpha)h_i \cdot \left(\alpha\rho_i C_i(\underline{\sigma}) + (1-\alpha)h_i(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right) - w\rho_i\left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)(1-\alpha)h_i \cdot \alpha C_i(\underline{\sigma})}{\left[\alpha\rho_i C_i(\underline{\sigma}) + (1-\alpha)h_i\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right]^2}$$

$$= \lambda$$

$$\frac{w\left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)(1-\alpha)^2 h_i^2\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)}{\left[\alpha\rho_i C_i(\underline{\sigma}) + (1-\alpha)h_i\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right]^2} = \lambda$$

λ: $\rho_i + \rho_j = 1$

So

$$\frac{h_i^2\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)}{\left[\alpha\rho_i C_i(\underline{\sigma}) + (1-\alpha)h_i\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right]^2} = \frac{h_j^2\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right)\left(C_j(\overline{\sigma}) - C_j(\underline{\sigma})\right)}{\left[\alpha\rho_j C_j(\underline{\sigma}) + (1-\alpha)h_j\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right)\right]^2}$$

And solve this:

$$\frac{h_i\sqrt{\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\left(C_i(\overline{\sigma}) - C_i(\underline{\sigma})\right)}}{\alpha\rho_i C_i(\underline{\sigma}) + (1-\alpha)h_i\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)} = \frac{h_j\sqrt{\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right)\left(C_j(\overline{\sigma}) - C_j(\underline{\sigma})\right)}}{\alpha\rho_j C_j(\underline{\sigma}) + (1-\alpha)h_j\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right)}$$

$$h_i\sqrt{\left(C_i(\overline{\sigma})^2 - C_i(\underline{\sigma})^2\right)}\left[\alpha(1-\rho_i)C_j(\underline{\sigma}) + (1-\alpha)h_j\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right)\right]$$

$$= h_j\sqrt{\left(C_j(\overline{\sigma})^2 - C_j(\underline{\sigma})^2\right)}\left[\alpha\rho_i C_i(\underline{\sigma}) + (1-\alpha)h_i\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)\right]$$

$$h_i\sqrt{\left(C_i(\overline{\sigma})^2 - C_i(\underline{\sigma})^2\right)}\left[\alpha C_j(\underline{\sigma}) + (1-\alpha)h_j\left(C_j(\overline{\sigma}) + C_j(\underline{\sigma})\right)\right] - h_j\sqrt{\left(C_j(\overline{\sigma})^2 - C_j(\underline{\sigma})^2\right)}\ (1-\alpha)h_i\left(C_i(\overline{\sigma}) + C_i(\underline{\sigma})\right)$$

$$= h_j\sqrt{\left(C_j(\overline{\sigma})^2 - C_j(\underline{\sigma})^2\right)}\ \alpha\rho_i C_i(\underline{\sigma}) + h_i\sqrt{\left(C_i(\overline{\sigma})^2 - C_i(\underline{\sigma})^2\right)}\ \rho_i C_j(\underline{\sigma})$$

$$\rho_i\left(h_j\sqrt{\left(C_j(\overline{\sigma})^2-C_j\left(\underline{\sigma}\right)^2\right)}\,\alpha C_i\left(\underline{\sigma}\right)+h_i\sqrt{\left(C_i(\overline{\sigma})^2-C_i\left(\underline{\sigma}\right)^2\right)}\,\alpha C_j\left(\underline{\sigma}\right)\right)$$

$$=\ \alpha C_j\left(\underline{\sigma}\right)h_i\sqrt{\left(C_i(\overline{\sigma})^2-C_i\left(\underline{\sigma}\right)^2\right)}$$

$$+\,(1-\alpha)h_ih_j\left[\sqrt{\left(C_i(\overline{\sigma})^2-C_i\left(\underline{\sigma}\right)^2\right)}\left(C_j(\overline{\sigma})+C_j\left(\underline{\sigma}\right)\right)-\sqrt{\left(C_j(\overline{\sigma})^2-C_j\left(\underline{\sigma}\right)^2\right)}\left(C_i(\overline{\sigma})+C_i\left(\underline{\sigma}\right)\right)\right]$$

So,

$$\rho_i^*=\frac{\alpha h_iC_j\left(\underline{\sigma}\right)\sqrt{\left(C_i(\overline{\sigma})^2-C_i\left(\underline{\sigma}\right)^2\right)}+(1-\alpha)h_ih_j\left[\sqrt{\left(C_i(\overline{\sigma})^2-C_i\left(\underline{\sigma}\right)^2\right)}\left(C_j(\overline{\sigma})+C_j\left(\underline{\sigma}\right)\right)-\sqrt{\left(C_j(\overline{\sigma})^2-C_j\left(\underline{\sigma}\right)^2\right)}\left(C_i(\overline{\sigma})+C_i\left(\underline{\sigma}\right)\right)\right]}{\alpha h_iC_j\left(\underline{\sigma}\right)\sqrt{\left(C_i(\overline{\sigma})^2-C_i\left(\underline{\sigma}\right)^2\right)}+\alpha h_jC_i\left(\underline{\sigma}\right)\sqrt{\left(C_j(\overline{\sigma})^2-C_j\left(\underline{\sigma}\right)^2\right)}}$$

List of equity options in empirical investigation

Symbols of Underlying Stock in the Option Dataset				
AIG	DELL	IBM	NOK	WMB
AMAT	DRYR	INTC	NXTL	WMT
AMGN	EBAY	JNJ	ORCL	XMSR
AMZN	EMC	JPM	PFE	XOM
AOL	EP	KLAC	QCOM	YHOO
BAC	F	KO	RX	
BSX	GE	MMM	S	
C	GM	MO	TXN	
CPN	HD	MRK	TYC	
CSCO	HPQ	MSFT	UPS	

VPIN Estimation under different parameter combinations

Average VPIN under Different Parameter Combinations (V=avg daily volume)				
OTM Puts	ATM Puts	ATM Calls	OTM Calls	Parameter Choice
0.775	0.659	0.556	0.686	**V/2, 30**
0.776	0.659	0.556	0.686	**V/2, 35**
0.776	0.659	0.556	0.686	**V/2, 40**
0.702	0.578	0.470	0.606	**V, 30**
0.703	0.577	0.470	0.606	**V, 35**
0.703	0.577	0.470	0.607	**V, 40**
0.662	0.520	0.415	0.555	**1.5V, 30**
0.662	0.520	0.416	0.557	**1.5V, 35**
0.662	0.520	0.416	0.557	**1.5V, 40**

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